The image shows the main facade of the TUHH building, a large red brick structure with a central arched entrance and two modern glass wings on either side. The sky is blue with light clouds.

High Quality Questions for E-Assessment in Mathematics

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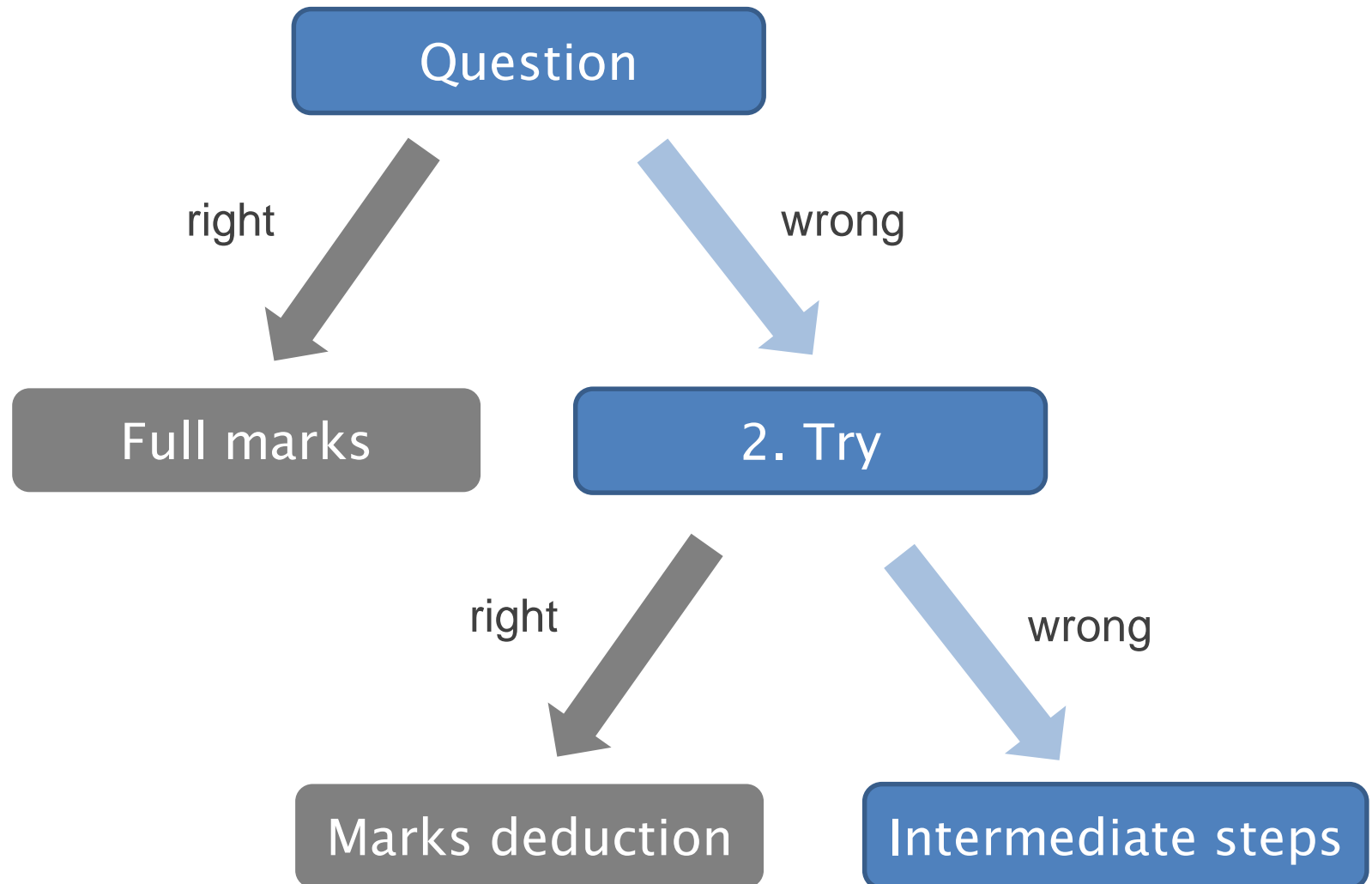
Adaptive Questions

Randomisation

Proof Puzzles

Survey Results

Adaptive Questions



- Intermediate steps according to a classical solution
- Sample solutions after every intermediate step
- Handling of follow-up errors

Determine the set of solutions in \mathbb{R}^3 of the system of linear equations

$$\begin{pmatrix} 2 & 2 & 2 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Bring the system of linear equations in a row echelon form.

A row echelon form of the system of linear equations is

$$\left(\begin{array}{ccc|c} 2 & 2 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{array} \right).$$

Determine the free variables.

By passing the variable x_3 to the right-hand side, we get the system of linear equations:

$$\begin{pmatrix} 2 & 2 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2x_3 \\ x_3 \end{pmatrix}.$$

Determine the set of solutions depending on the variable x_3 .

Pros	Cons
Efficiently check follow-up errors	Fixed solution path
Fair & transparent assessment	Enter intermediate results
Higher learning success	Higher creation effort

- Individual tasks make fraud attempts more difficult
- Challenge: comparable level of difficulty
- Appropriate assessment schemes/algorithms needed

Randomisation via Reverse Engineering:

Determine the set of solutions in \mathbb{R}^3 of the system of linear equations

$$\begin{pmatrix} 2 & 2 & 2 \\ 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

- Choose the entries of the LU decomposition randomly:

$$\begin{pmatrix} 2 & 2 & 2 \\ 2 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \mathbf{1} & 1 \end{pmatrix} \begin{pmatrix} \mathbf{2} & \mathbf{2} & \mathbf{2} \\ 0 & \mathbf{1} & \mathbf{-1} \end{pmatrix}$$

- Control over the complexity of the calculation steps

Proof Puzzles

- Assess proofs electronically
- Sort via drag'n'drop
- Grading algorithm
 - Edit distance
 - Works even with several sample solutions

Pros	Cons
Quite intuitive input	Fixed structure
Automatic grading	Only short proofs
Assess logical reasoning	Sentence structure exploitable

Let $n \in \mathbb{Z}$. Find a proof for the statement

"If n is even, then n^2 is also an even number".

[4,2,1,5,6]



Proof:

Let $n \in \mathbb{Z}$ be an even number.

4

This means, it exists a number $p \in \mathbb{Z}$ such that

2

$n = 2p$ holds.

1

This implies $n^2 = (2p)^2 = 2(2p^2)$.

5

Hence, it holds $n^2 = 2m$ with $m := 2p^2 \in \mathbb{Z}$.

6

Not used:

Consequently, n^2 is even.
Consequently, n^2 is even.

3

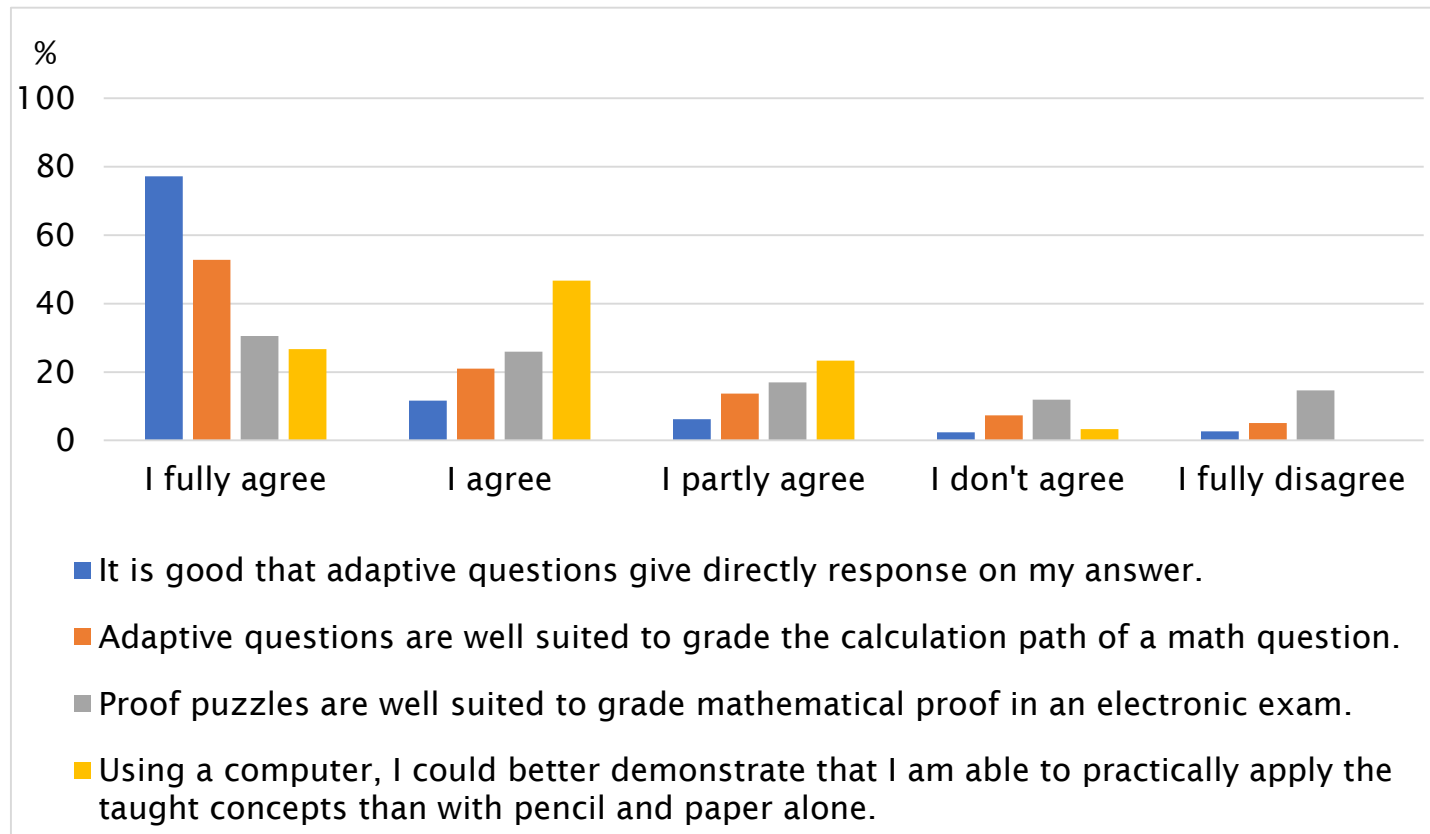
$n = p^2$ holds.

8

Suppose that $n \in \mathbb{Z}$ is odd.

7





Results of surveys conducted in the courses Mathematics for engineers (questions 1-3, N=415) and Stochastics (question 4, N=41).



Thank you for your attention!