



MatRIC Centre for Research,
Innovation and Coordination
of Mathematics Teaching



Probability and Statistical Methods: Assessing Knowledge and Competencies (pre and during Covid19 pandemic) – case study at ISEC

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RULES Math

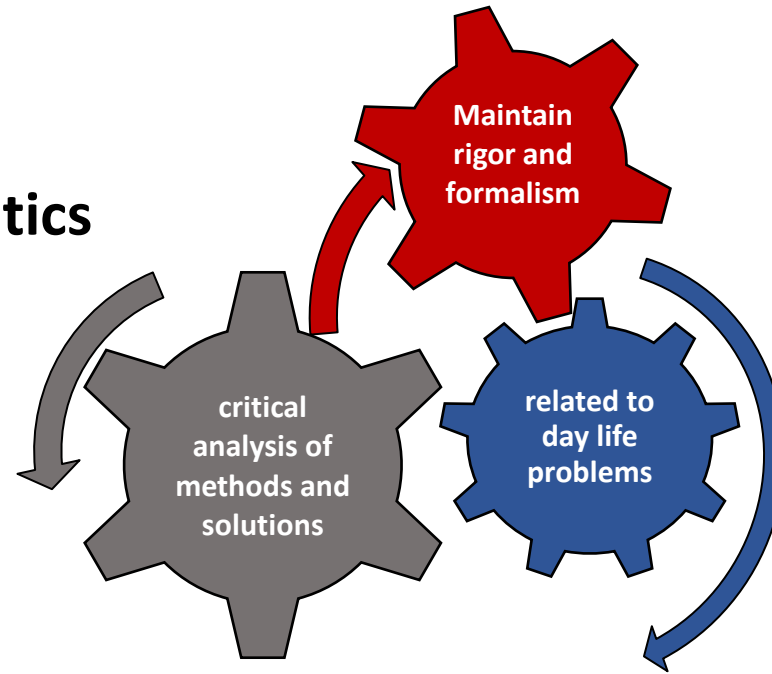




- Outline

- Introduction
- RULES_MATH Project
- Course unit characterization
- Activities and Assessment (Pre and during Covid 19)
- Results
- Conclusions
- Acknowledgments
- References

Teaching mathematics



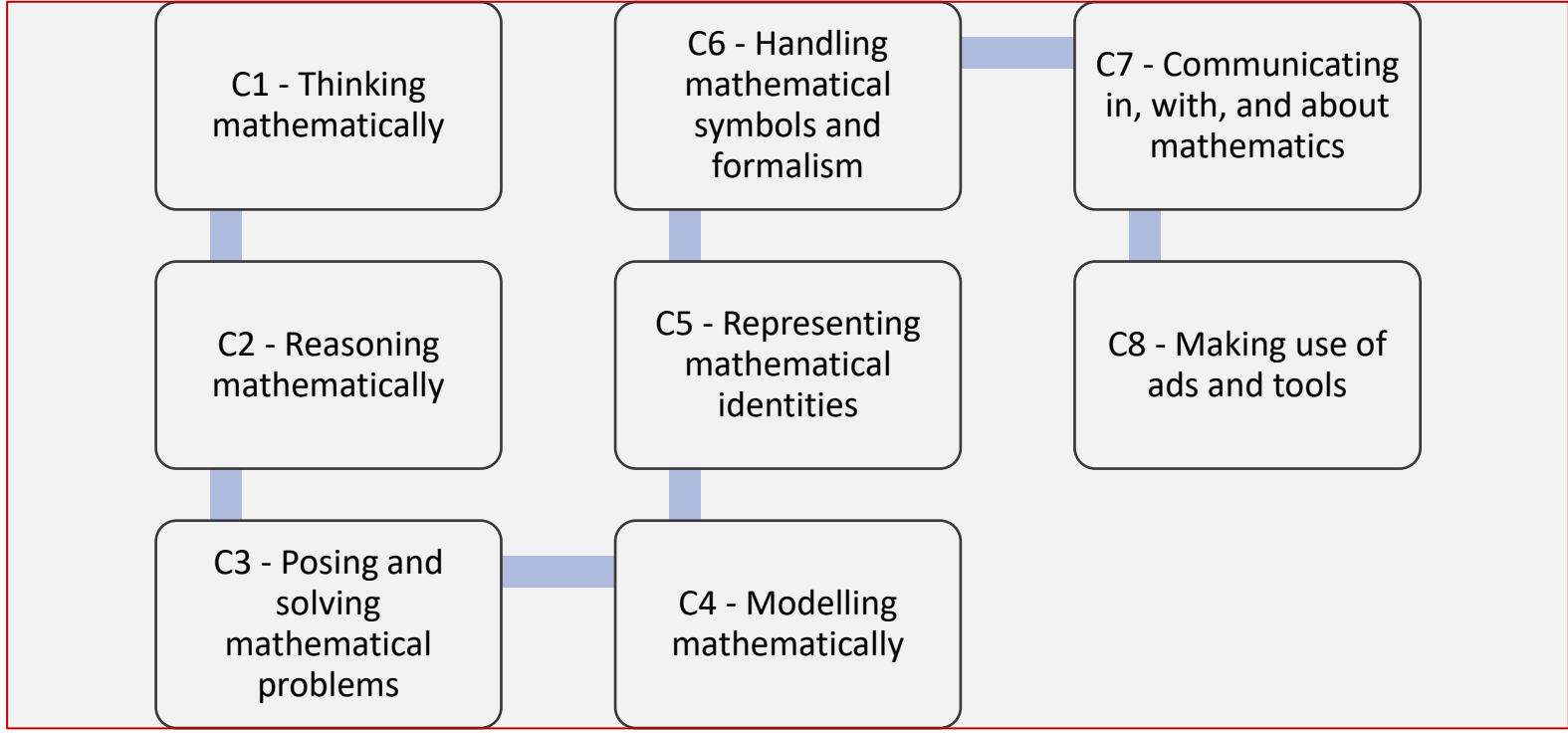
While keeping students engaged and active in their learnings

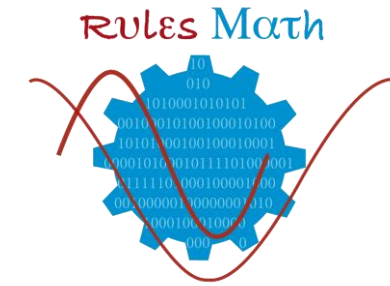
Niss, M., (2003) Mathematical Competencies and the Learning of Mathematics: The Danish KOM Project.

Alpers, B. et al, (2013) A framework for mathematics curricula in engineering education, SEFI,. Available online at: <http://sefi.htw-aalen.de/>

2003 – KOM Project – Kompetencer Og Matematiklæring – Identification of skills and mathematical competencies

European Society for Engineering Education
Europäische Gesellschaft für Ingenieur-Ausbildung
Société Européenne pour la Formation des Ingénieurs





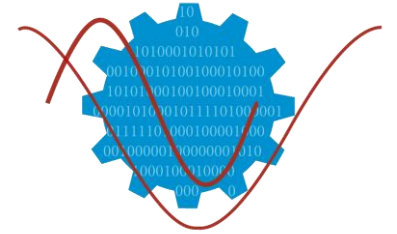
RULES_MATH Project, Project Erasmus+ 2017-1-ES01-KA203-038491” New Rules for Assessing Mathematical Competencies “. <https://www.researchgate.net/project/New-Rules-for-assessing-Mathematical-Competencies>.

Project live duration ended formally in 2020 but we desire their findings to continue in the future!

- ✓ On the completion of the project, the expected result was to achieve an agreement about the standards for assessing mathematical competencies in engineering degrees, *i.e.*, to get new rules to assess and also to have materials and resources to assess mathematical competencies and disseminate them.

Pre-COVID 19 and During COVID 19

RULES Math

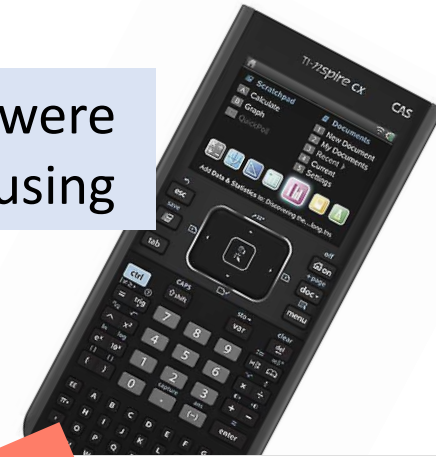


Course unit

1st Part – Probability theory and distributions (discrete and continuous)

2nd Part – Statistics: sampling and inference

All calculus were performed using



Time with students (~ 120 per year)

4 hours per week + **6 office hours** per week (most of them only appear 1 week before assessment tests)

SRICTLY ONLINE in 2020/2021 academic year

```
1 x <- rnorm(1000)
2 hx <- hist(x, breaks=100, plot=FALSE)
3 plot(hx, col=ifelse(abs(hx$breaks) < 1.669, 4, 2))
4
```

Result

script main.r

Histogram of x

Frequency

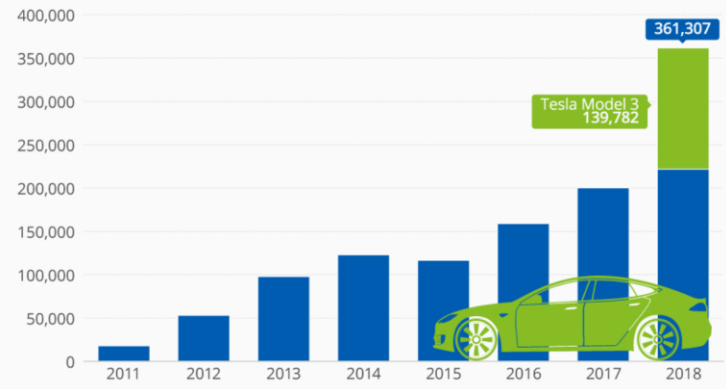
x

The screenshot shows an R console with three lines of code: generating 1000 random numbers, creating a histogram object, and plotting it with two colors. The resulting histogram shows a normal distribution of data points. A large red diagonal banner is overlaid on the screenshot.

Calculus were performed using calculators machines and also R-platforms freely accessible online

Tesla Model 3 Boosts Electric Cars Sales

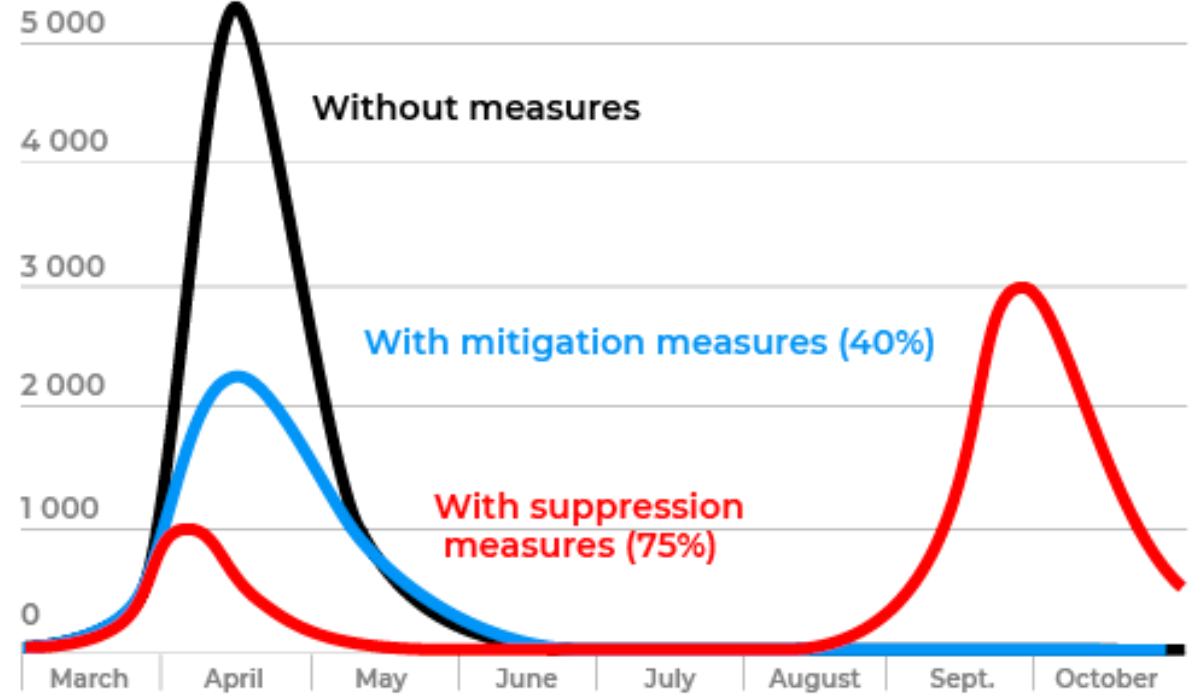
Estimated plug-in electric vehicle sales in the United States*



* incl. plug-in hybrids
Source: InsideEVs

statista

ACCORDING TO THIS MODEL,
the peak of infection has passed:

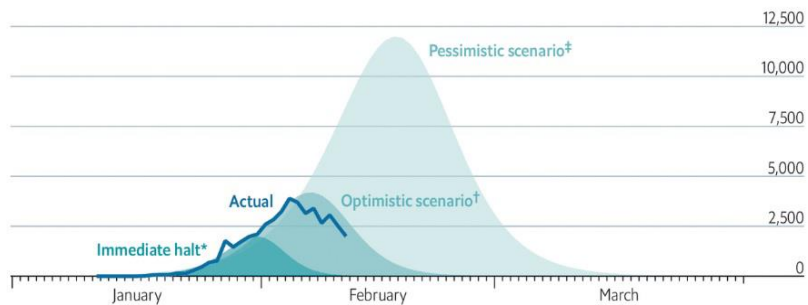


Source: Gabriela Gomes, epidemiology specialist mathematician at Liverpool School of Tropical Medicine, UK.

PUBLICICO

Breathe a bit more easily

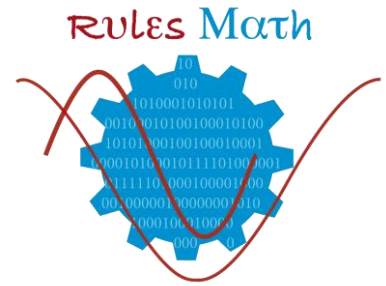
China, new confirmed cases of Wuhan coronavirus, 2020



Sources: Cheng-Chih Hsu, National Taiwan University; National Health Commission of China
*Hypothetical model of Wuhan quarantine immediately stopping the spread
†Based on estimates by Zhong Nanshan, Guangzhou Institute of Respiratory Disease
‡Based on estimates by Jonathan Read, Lancaster University

The Economist

Nevertheless, an exam is demanded!



- = very important
- = medium important
- = less importante

and SP_{nm} stands for Statistics and Probability/section/sub section of the Core Level 1, [1].

Table 1: Learning outcomes with degree of coverage of competencies involved in this assessment activity.

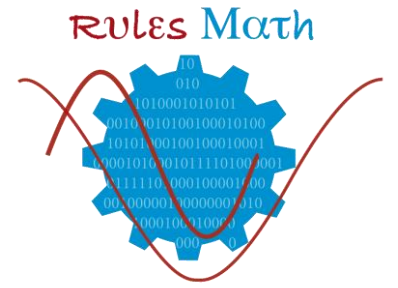
		C1	C2	C3	C4	C5	C6	C7	C8
Statistics and Probability									
SP3	3. Simple probability								
SP31	Interpret probability as a degree of belief								
SP32	Understand the distinction between a priori and a posteriori probabilities								
SP33	Use a tree diagram to calculate probabilities								
SP34	Know what conditional probability is and be able to use it (Bayes' Theorem)								

Statistical Methods
Core Level 1 - SEFIMWG

Competencies

Competencies covered

- Thinking mathematically
- Reasoning mathematically
- Posing and solving mathematical problems
- Modelling mathematically
- Representing mathematical identities
- Handling mathematical symbols and formalism
- Communicating in, with, and about mathematics
- Making use of aids and tools

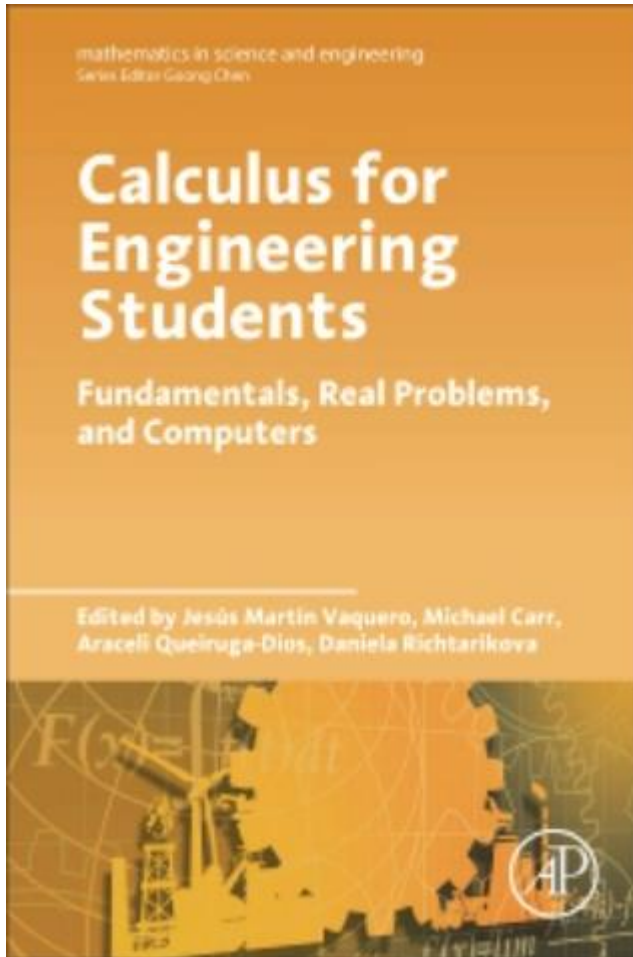
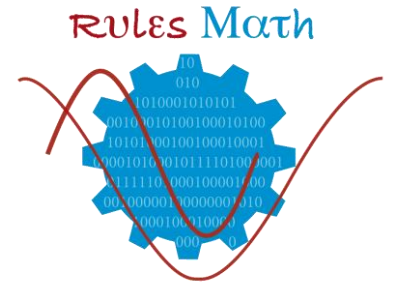


RULES_MATH project partners' working groups have developed a set of "Guide for a Problem" in the different areas of Mathematics that are intended to provide some examples of proposed forms of assessment and competence-based activities. The materials are available at <https://rules-math.com/> and all project partners applied them to different students from different courses at their institutions.



Calculus for Engineering Students. <https://doi.org/10.1016/B978-0-12-817210-0.00017-5>

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<https://rules-math.com/>



HOME WELCOME PARTNERS OBJECTIVES INTELLECTUAL OUTPUTS DISSEMINATION TRAINING

- 7. Communicating in, with, and about Mathematics.
- 8. Making use of aids and tools for mathematical activity.

We have also included the objectives, contents, learning outcomes from the Core Level 1, and a possible exam for some specified LO. This exam could be used as a small project (miniproject) that can be worked either in groups or individually or both options: start working individually and then share and discuss the results in a group.

Assessment standards defined during this project for assessing mathematical competencies are the following:

1. Multiple-Choice Questions.
2. Questions or «classical» problems.
3. Projects.

All together make up the way to assess competences. One of them is not enough, but the three of them make the best option.

These user guides and separated files for each discipline are available at the moodle platform of the project (<https://studium.usal.es/>) and also included in the book of the project:

«New Rules for Assessing Mathematical Competencies: USER GUIDE»
Snezhana Gocheva-Ilieva and Araceli Queiruga-Dios Editors
© P. P. Hilendarski University Publishing House
ISBN: 978-619-202-575-5

NEW RULES FOR ASSESSING MATHEMATICAL COMPETENCIES || RULES_MATH
PROJECT IDENTIFICATION: 2017-ES01-KA203-038491

KA2 - Cooperation for Innovation and the Exchange of Good Practices KA203 - Strategic Partnerships for higher education.

Co-funded by the

ASSESSMENT - 2 tests and/or final exam

Structure

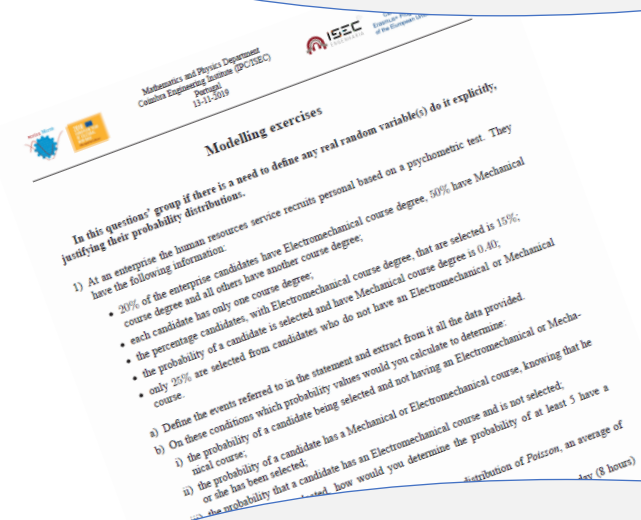
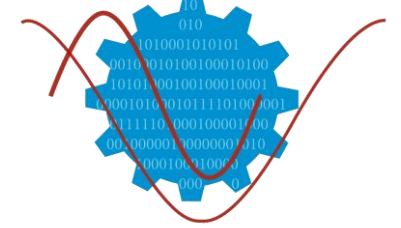
a set of multiple-choice questions

a problem-based set of questions

set of development questions

Pre-COVID 19

Rules Math



Suppose that you are playing a computer game where the objective is to destroy a section of a railway line piloting an airplane (Figure 1). You are an element of the airplane crew that receives the order to destroy and you are the one that has to do the calculations and give the instructions to your fellow colleagues.



Figure 2. Railway section to be destroyed

2. To the crew of an airplane is assigned the task of destroying a section of railway line. The crew's aim statistics reveal that the variable X , distance, in meters, from the point of impact of a bomb to the targeted line, follows a Normal distribution with standard deviation $6m$. It is considered equally probable that the bomb falls to one side (for which it is agreed to take $X > 0$) or to the other (for which it is agreed to take $X < 0$) of the line. The mission is considered fulfilled if at least one bomb hits the target, that is, if it "falls to less than $1m$ of the line".
 - a. Justify that X has mean value equal to zero.
 - b. What is the probability of the bomb hits the target?
 - c. Characterize the distribution function of the random variable $Y = \text{"Number of bombs that hit the target, supposing that } n \text{ bombs are launched (} n \in \mathbb{N} \text{)"}$.
 - d. Suppose that $n > 20$ and determine, using the approximate distribution of Y , the minimum number of bombs that is necessary to launch in order to have a successful mission with probability greater than 0.95 .

3. Suppose the weight, W , of each bomb is a random variable with Normal distribution with mean 65 kg and standard deviation 3 kg and that the maximum load supported by the mission plane is 3 tones .

ASSESSMENT – Classes exercises and 1 test and/or final exam

During COVID 19

Structure

a set of multiple-choice questions

Question 10
Correct
Mark 1.50 out of 1.50

Sejam X e Y duas variáveis aleatórias independentes, normalmente distribuídas com valores esperados 3.1 e 4.5 e com desvios padrão 2 e 3, respectivamente. A probabilidade da variável aleatória $T = 2X + Y$ não exceder 27 é melhor aproximada por (nos cálculos intermédios use pelo menos 2 casas decimais):

- a. 0.9994
- b. 0.9932
- c. 0.9997
- d. 0.8810

Your answer is correct.
The correct answer is: 0.9994

Make comment or override mark

Response history

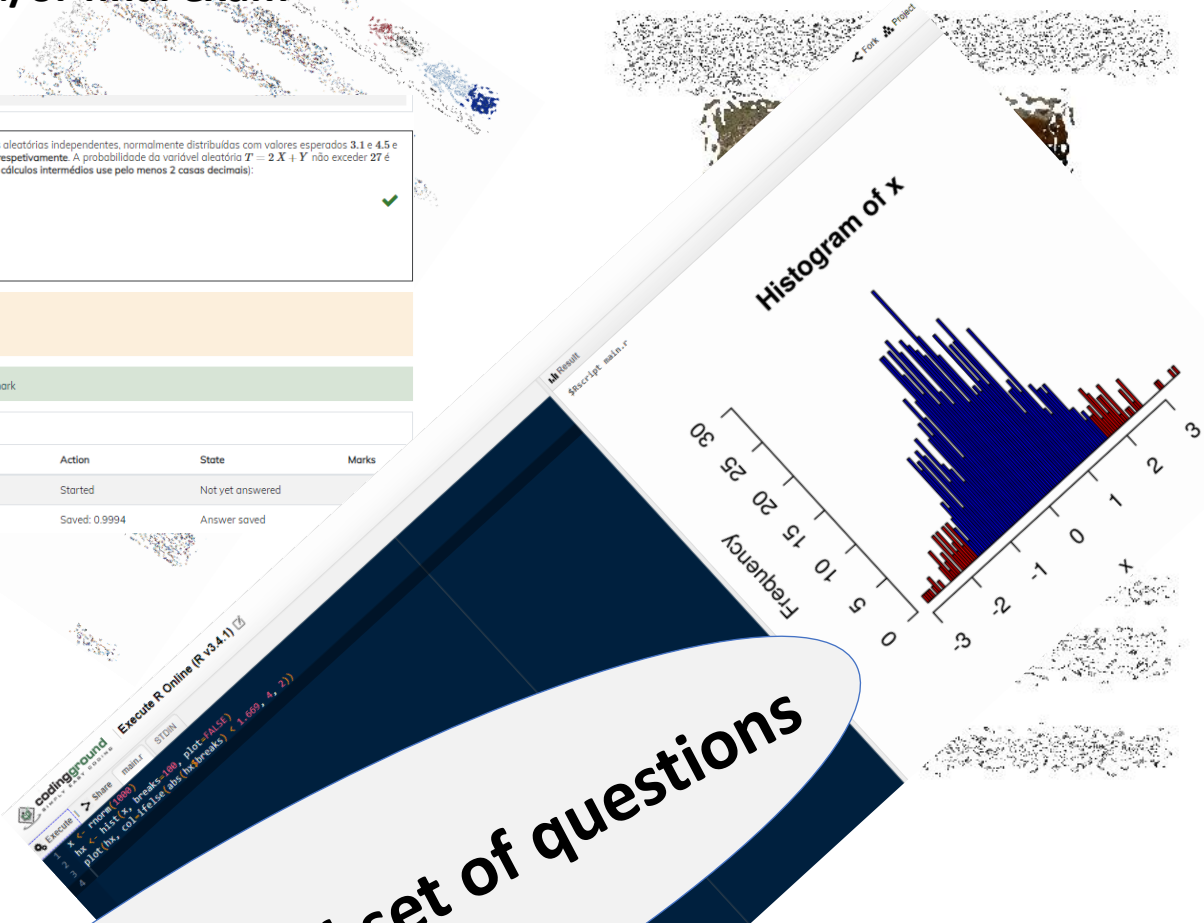
Step	Time	Action	State	Marks
1	17/02/21. 09:30	Started	Not yet answered	
2	17/02/21. 11:16	Saved: 0.9994	Answer saved	

Question 4
Not answered
Marked out of 2.00

O número de encomendas de um certo produto que chegava diariamente a uma empresa, antes da Covid-19, era bem ajustado por uma Poisson de média 3 (atualmente, sabemos que a situação é bem diferente). Determine a probabilidade de, naquela época, serem efetuadas mais de 10 encomendas em 5 dias.

set of development questions

a problem-based set of questions





Question categories for 'Course: 912313 - M

- Default for 912313_Mét_EM_2021 (0) The default category for questions shared

- probRecurso2021 (6)
- AditPoirec (7)
- estrectic (6)
- ParamPropPoirec (8)
- Discretasrec (8)
- Vetrec (4)
- normaditrec (2)
- estimvarec (4)
- estnormrec (4)
- Questão aula 10 (4)
- Questão aula 9 (4)
- Questão aula 7 e 8 (9)
- Questão aula nº6 (8)

Create a new question ...

Question
Question name / ID number
Q1discretasrec
A função de probabilidade de uma variável aleatória discreta X é dada pela tabela seguinte, onde a e b pertencem

Actions

Created by
First name / Surname / Date
Deolinda Rasteiro
16 February 2021, 10:38 PM

Last modified by
First name / Surname
Deolinda Rasteiro
16 February 2021, 10:38 PM

x	-2	-1	1	2	3	c.c.
$P(X=x)$	a	b	$1/4$	b	a	0

Sabendo que $P(X=-2) = 2 \times P(X=2)$, determine o valor de $P(X=4a+8b)$?
Q2discretasrec
A função de probabilidade de uma variável aleatória discreta X é dada pela tabela seguinte, onde a e b pertencem

x	-2	-1	0	1	2	c.c.
$P(X=x)$	a	b	$1/4$	b	a	0

Sabendo que $P(X=-2) = 2 \times P(X=1)$, determine o valor de $P(X=4a+8b)$?
Q3discretasrec
A função de probabilidade de uma variável aleatória discreta X é dada pela tabela seguinte, onde a e b pertencem

x	-2	-1	0	1	2	c.c.
$P(X=x)$	a	b	$1/4$	b	a	0

Sabendo que $P(X=-2) = 2 \times P(X=1)$, determine o valor de $P(X=4a+2b)$?
Q4discretasrec
A função de probabilidade de uma variável aleatória discreta X é dada pela tabela seguinte, onde a e b pertencem

x	-2	-1	1	2	3	c.c.
$P(X=x)$	a	b	$1/4$	b	a	0

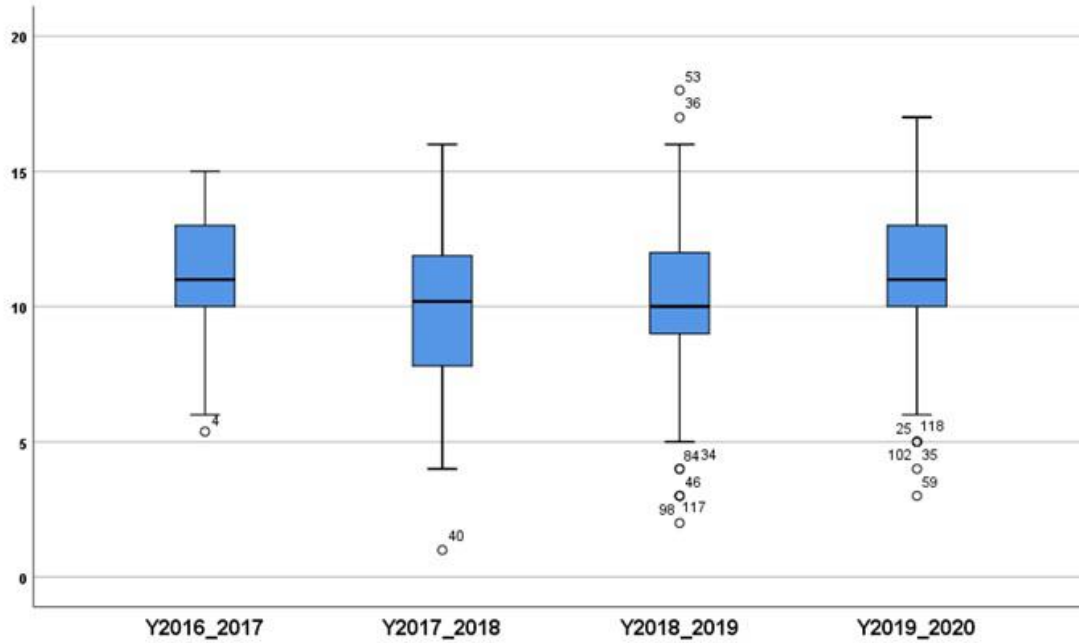
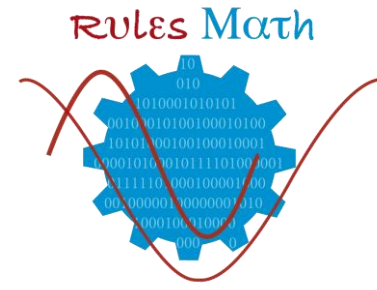
Sabendo que $P(X=-2) = 2 \times P(X=2)$, determine o valor de $P(X=4a+2b)$?
Q5discretasrec
A função de probabilidade de uma variável aleatória discreta X é dada pela tabela seguinte, onde a e b pertencem



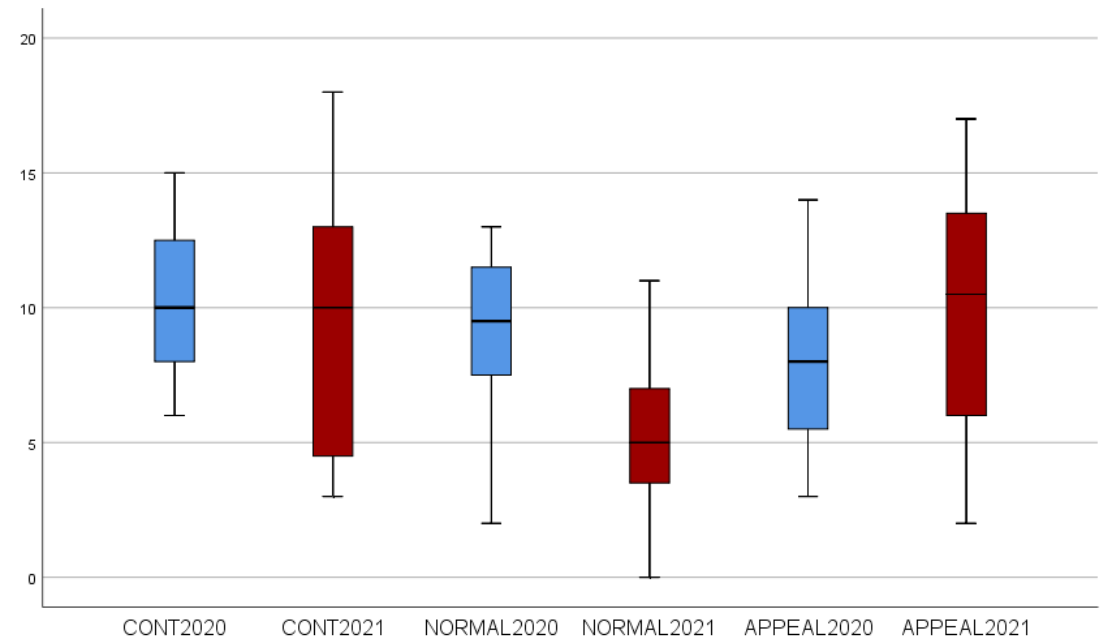
• During COVID 19

RESULTS

Pre-COVID 19 and During COVID 19



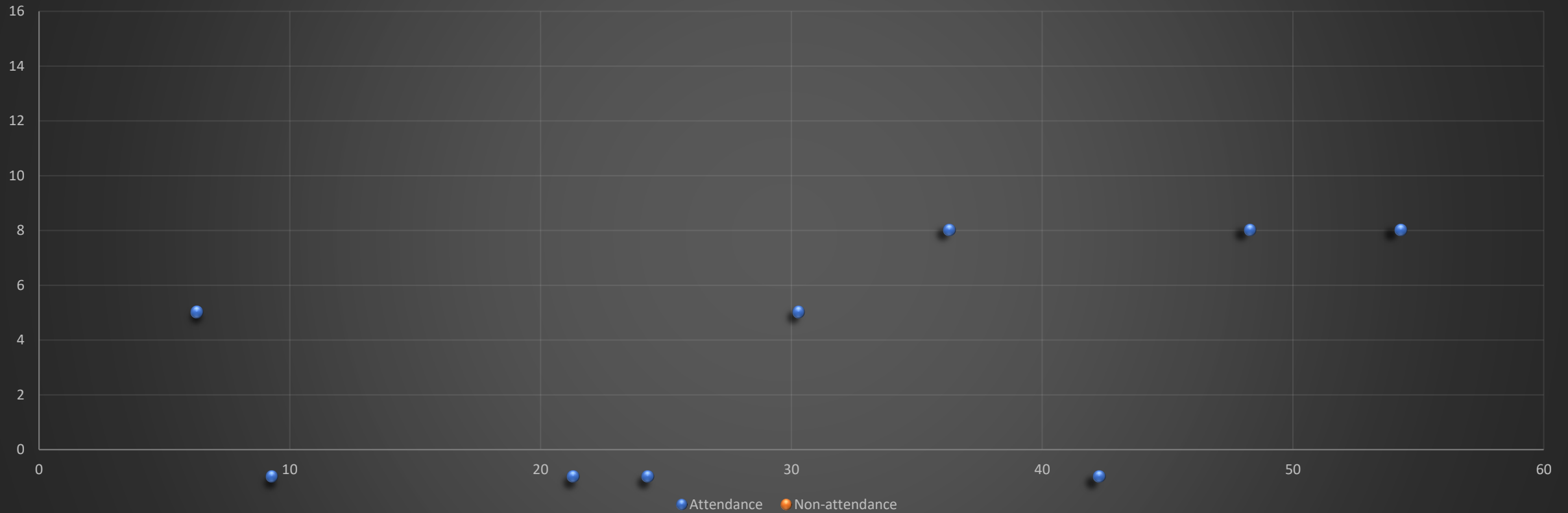
76.3% of students present at the assessment moments, had above 10 out of 20



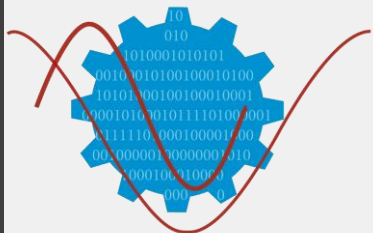
Competence-methodology empowers students with a better preparation to face and deal with real-life problems, students became more critics and analysed questions and solutions in a more professional way

RESULTS

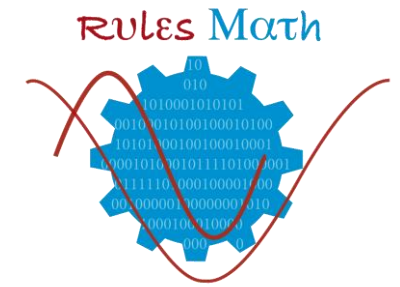
Grades/Attendance/Non-Attendance



RULES MATH



- Attending to 7, or more, classes out of 15



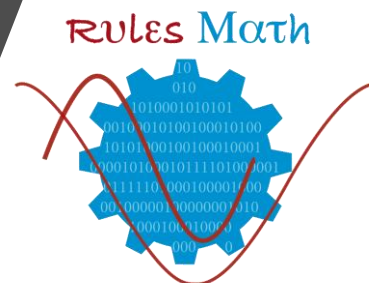
RESULTS

- Throughout the past decade we observed that students were increasing their absence to assessment.
- With competence-methodology we observe a generous setback on this situation.

	Valid N	Valid Percent	Missing N	Missing Percent	Total N	Total Percent
Y2016/2017	53	44.2%	67	55.8%	120	100%
Y2017/2018	57	47.5%	63	52.5%	120	100%
Y2018/2019	85	70.8%	35	29.2%	120	100%
Y2019/2020	72	60.0%	48	40.0%	120	100%
Y2020/2021	102	85.0%	18	15.0%	120	100%

CONCLUSIONS

- To know which competencies are less acquired by students and immediately solve those issues => make a continuous assessment probably during classes (this has been done in 2020/2021 year with good results)
- There exists a difference between questions of understanding, representing and modelling and questions that only require calculus and formulas and “mechanical work” and they must be addressed separately
- With competence-methodology students were more involved in learning and assessment moments
- With the correct assessment preparation, no big worries with cheating are needed
- Class attendance and student direct needs attendance improves learning quality and results
- **Initial student’s background level up must be taken care – TO DO LIST (urgent)**



Acknowledgement

Financial support of the Erasmus+ project 2017-1-ES01-KA203-038491 "New Rules for Assessing Mathematical Competencies" is gratefully acknowledged.

- [1]Alpers, B., et al., "A Framework for Mathematics Curricula in Engineering Education", Proceedings of SEFI MWG Seminar, 2013. Available at: <http://sefi.htw-aalen.de/>
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References

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