On embedding dynamic mathematical tools into computer-aided assessment systems

- *preliminary findings from a pilot study*

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Some background

- The first Calculus course for first year engineering students
  Functions
  - Comprehending the concept of a function and working with functions

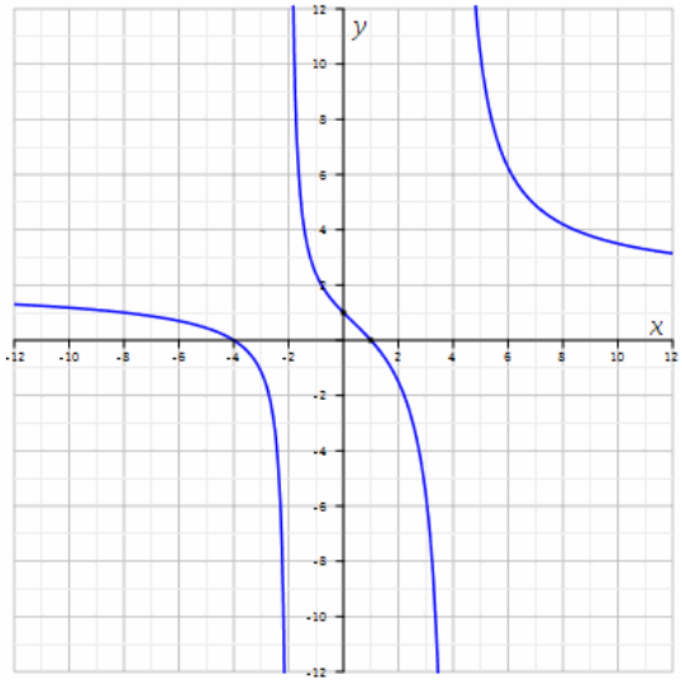
- Type of tasks: translation tasks and example generating tasks

- Digital tools
  - GeoGebra / Dynamic Mathematics Software (DMS)
  - Möbius / Computer-Aided Assessment (CAA) System

- Pilot study (in the autumn of 2020)
  Focus: to get a deeper understanding on
  - the interplay between these digital tools in designing different types of learning tasks, as well as
  - those prevailing student strategies when performing these tasks

Example 1 (Task 5a)

Below is the graph of a function g.



i) Use the graph to determine the function formula. Check your suggestion in GeoGebra before submitting it as an answer to the task.

Group agreed response

$g(x) =$

ii) Give a brief account of how you used the graph to determine the function formula.

Group agreed response

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Example 1 (Task 5a), Continued…

Category/Code	Group agreed response	Alternative formulation	Total # groups	
5A	$\frac{2(x+4)(x-1)}{(x+2)(x-4)}$	$\frac{2(x^2+3x-4)}{x^2-2x-8}$	46	(45.5 %)
5B	$\frac{2}{x+2}+\frac{8}{x-4}+2$	$2\left(\frac{1}{x+2}+\frac{4}{x-4}\right)+2$	17	(16.8 %)
5C	$\frac{10x+8}{(x+2)(x-4)}+2$		22	(21.8 %)
5D	$\frac{-x}{2+x}+\frac{2x}{x-4}+1$		2	(2.0 %)
5E	$\frac{2x^2+6x-8}{(x+2)(x-4)}$		1	(1.0 %)
None of the above (NA)			7	(6.9 %)
No response (NR)			6	(5.9%)
Total # groups			101	

Table 1: Major categories of the structure of the algebraic expressions presented as group response for Task 5a and summary of their respective number of occurrences.

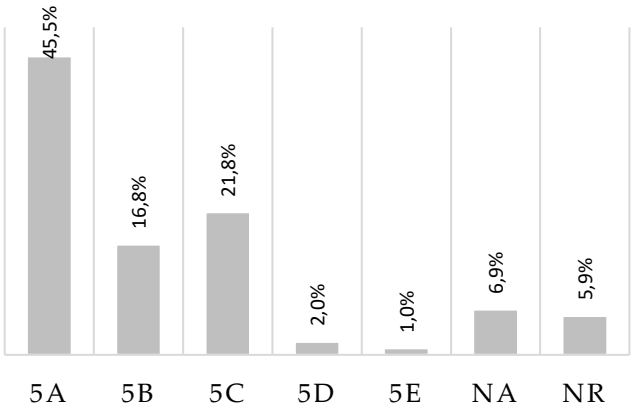
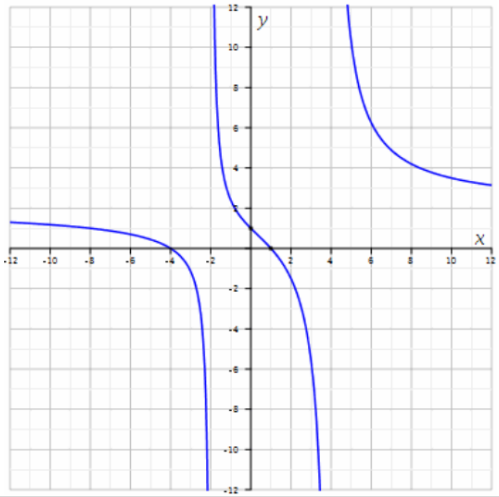


Figure 1. Graphic illustration (data from Table 1)

Example 1 (Task 5a), Continued…



Explanations - contain a reference to one or more of the following "key elements":

VA - the vertical asymptotes of the graph of g ($x = -2$ and $x = 4$)

HA - the horizontal asymptote of the graph of g ($y = 2$)

Zs - the x intercepts of the graph of g ($x = -4$ and $x = 1$)

1P - one additional point on the graph of g

GG - GeoGebra

ES - system of equations or two additional points on the graph of g

Key element	# groups	
VA	84	95.5 %
HA	47	53.4 %
Zs	44	50.0 %
1P	25	28.4 %
GG	15	17.0 %
ES	20	22.7 %
Total # of groups	88	

Table 2: Overview of "key elements" in the explanation.

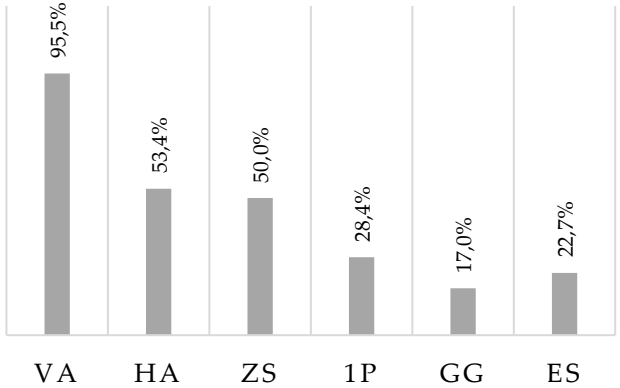


Figure 2. Graphic illustration (data from Table 2).

Example 1 (Task 5a), Continued…

A closer look: "key elements" versus structure of formula (or vice versa)

VA -	the vertical asymptotes of the graph of g ($x = -2$ and $x = 4$)
HA -	the horizontal asymptote of the graph of g ($y = 2$)
Zs -	the x intercepts of the graph of g ($x = -4$ and $x = 1$)
1P -	one additional point on the graph of g
GG -	GeoGebra
ES -	system of equations or two additional points on the graph of g

Formula / Key element	5A	5B	5C	5D	5E	Total
VA	45	16	20	2	1	84
HA	15	16	15	0	1	47
Zs	44	0	0	0	0	44
1P	22	3	0	0	0	25
GG	5	3	6	1	0	15
ES	0	7	10	2	1	20
# Groups	46	17	22	1	1	88

Table 3: Overview of "key elements" in the explanation in relation to the structure of the formula presented as group response.

Reminder

5A	5B	5C	5D	5E
$\frac{2(x+4)(x-1)}{(x+2)(x-4)}$	$\frac{2}{x+2}+\frac{8}{x-4}+2$	$\frac{10x+8}{(x+2)(x-4)}+2$	$\frac{-x}{2+x}+\frac{2x}{x-4}+1$	$\frac{2x^2+6x-8}{(x+2)(x-4)}$

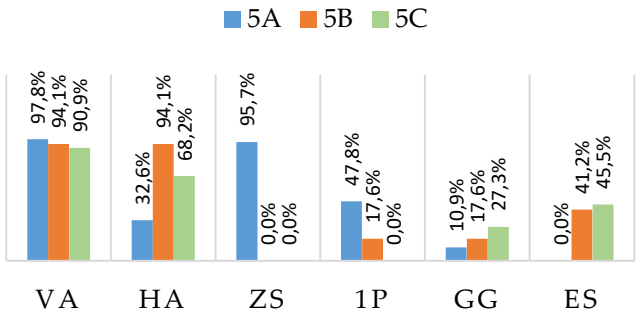


Figure 3. Key elements vs. structure of formula (data from Table 3).

Example 2 (Task 7)

Give examples of two different functions, f and g, both of which have

- two vertical asymptotes, $x = -6$ and $x = 3$, as well as
- a horizontal asymptote, $y = 2$.

Note:

- Group members may have received different asymptotes.
- Check in GeoGebra if your suggested functions really have the given asymptotes.

Individual response:

$f(x) =$

$g(x) =$

Example 2 (Task 7), Continued…

Category	Individual response (a and b constants, to be determined)	Total
7A	$\frac{2x^2+ax+b}{(x+6)(x-3)}$	31 (13.3 %)
7B	$\frac{a}{x+6}+\frac{b}{x-3}+2$	54 (23.2 %)
7C	$\frac{ax+b}{(x+6)(x-3)}+2$	137 (58.8 %)
7D	$\frac{x^2+ax+b}{(x+6)(x-3)}+1$	5 (2.1 %)
No response		6 (2.6 %)
Total (# indiv. resp)		233

Table 4: Major categories, structure of expressions for Task 7, individual response and summary of their respective number of occurrences.

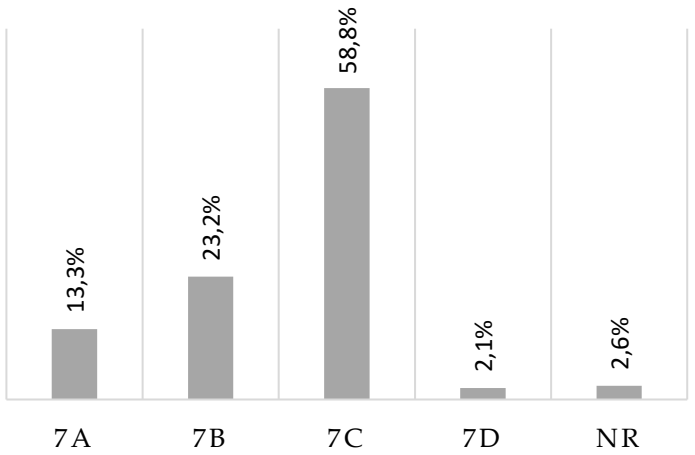


Figure 4. Graphic illustration (data from Table 4).

Reminder:
Give examples of two different functions, f and g, both of which have

- two vertical asymptotes, $x = -6$ and $x = 3$, as well as
- a horizontal asymptote, $y = 2$.

Group resp. / Individual resp.	5A	5B	5C	5D	5E	Total	
7A	23	2	3	0	3	31	(13.3%)
7B	16	30	8	0	0	54	(23.2%)
7C	75	14	43	5	0	137	(58.8%)
7D	4	1	0	0	0	5	(2.1%)
No response	4	0	2	0	0	6	(2.6%)
Total	122	47	56	5	3	233	

Table 5: Overview of the structure of formula (Task 7) - individual response in relation to the structure of formula (Task 5) presented as group response.

Reminder

5A	5B	5C
$\frac{2(x+4)(x-1)}{(x+2)(x-4)}$	$\frac{2}{x+2}+\frac{8}{x-4}+2$	$\frac{10x+8}{(x+2)(x-4)}+2$

7A	7B	7C
$\frac{2x^2+ax+b}{(x+6)(x-3)}$	$\frac{a}{x+6}+\frac{b}{x-3}+2$	$\frac{ax+b}{(x+6)(x-3)}+2$

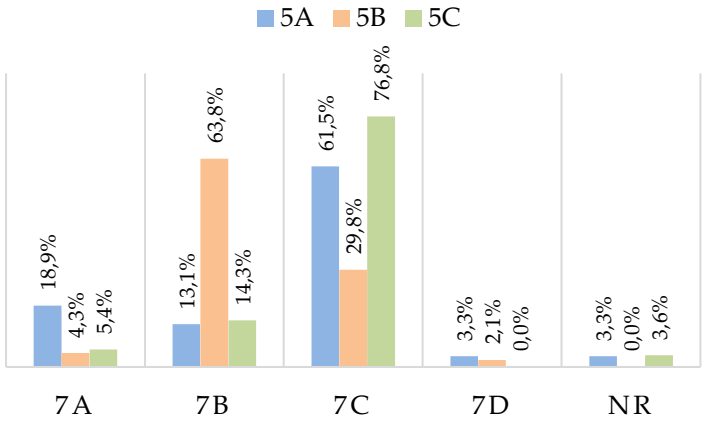


Figure 5. Group response vs. individual response (data from Table 5).

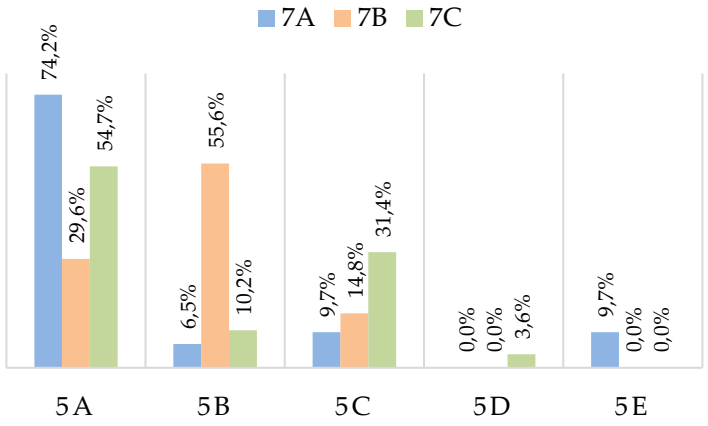


Figure 6. Individual response vs. group response (data from Table 5).

Example 2 (Task 7), Continued…

Students' strategies (for producing the second function in Task 7)

- S1: Adding or multiplying with a constant
- S2: Algebraically manipulating the denominator
- S3: Changing the numerator in another way than in S1, e.g. adding a polynomial
- S4: Multiplying numerator and denominator by x or a constant
- S5: Manipulating the expression to receive a common denominator and changing a constant
- S6: Switching to another type of formula

Formula / Strategy	7A	7B	7C	7D	Total	
S1	17	51	118	3	189	71.6 %
S2	2	0	9	0	11	4.2 %
S3	7	0	10	1	18	6.8 %
S4	8	5	5	0	18	6.8 %
S5	0	0	2	0	2	0.8 %
S6	0	0	5	1	6	2.3 %
No answer	2	0	1	0	3	1.1 %
Same ans.	1	1	0	0	2	0.8 %
SA rewritten	8	5	2	0	15	5.7 %
Total	45	62	152	5	264	

Table 6: Overview of students' strategies used to produce a second example in Task 7.

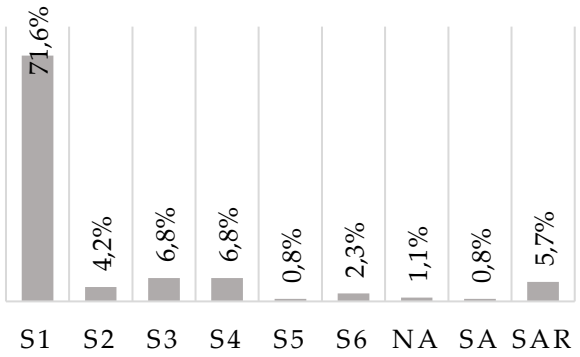


Figure 7. Strategies in Task 7, graphic illustration (data from Table 6).

7A	7B	7C
$\frac{2x^2+ax+b}{(x+6)(x-3)}$	$\frac{a}{x+6}+\frac{b}{x-3}+2$	$\frac{ax+b}{(x+6)(x-3)}+2$

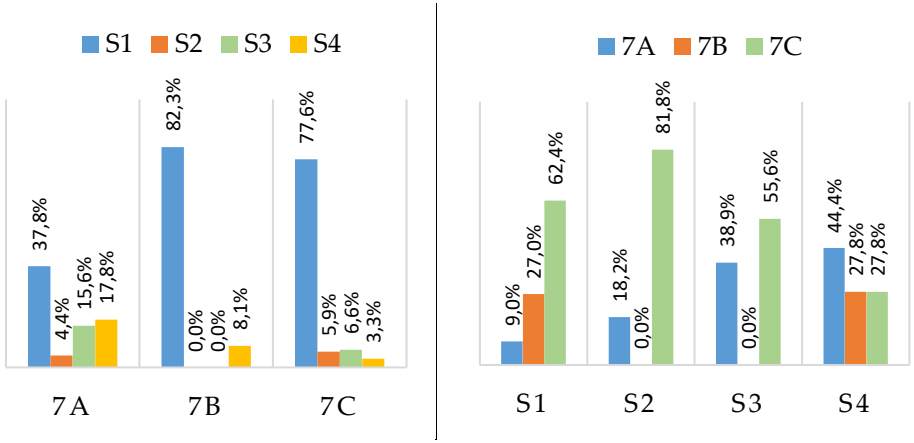


Figure 8. Strategies in Task 7. Closer look (data from Table 6).

Concluding remark

We continue to explore the potential in utilizing the adaptive features of the CAA system

Two examples

1) Instead of asking for an explanation, we could ask students to declare the explanation elements used by choosing among various suggested options. Depending on their response, they will receive different (adapted) feedback. For example, if a student have not used the horizontal asymptote, then it would be imperative to ask the student to solve a new task with this new strategy.

2) If a student use a correct formula, although without using the intended key ideas, the feedback could be something like: “Great, the answer is correct. However, another correct answer could be “like this…”. How do you think a student who came up with this answer has been reasoning? Now, use this strategy to provide an example of a function with the following…

Thank you for your attention.

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