

# Mathematical Reasoning in Engineering Statics

Burkhard Alpers

*Department of Mechanical Engineering, Aalen University*

## Abstract

Mathematical reasoning is one of eight mathematical competencies that are stated in the curriculum document of SEFI's Maths Working Group (Alpers et al. 2013). This was elaborated in more detail for a practice oriented study course in mechanical engineering in (Alpers 2012) based on the experience of the author with the kind of reasoning needed in such a course. Such experience should be substantiated by systematic investigation into the reasoning to be found in application subjects, particularly in theory development in such subjects. In this contribution, I look at engineering statics, one of the first and most basic modules of a study course in mechanical engineering. I analyse a few basic chapters of a textbook that is widely used in Germany (Gross et al. 2016, 13. Edition) and that has also been translated into English (Gross et al. 2013).

I will first elaborate on the concept of “mathematical reasoning” as it is discussed in literature in order to make available some categories that can be used for analysis. Moreover, I will present a few results on differences between mathematical practices in mathematics and other subjects. Then follows a presentation of the method of investigation and the main results. The contribution closes with a discussion of potential consequences for the mathematical education of engineers.

## References

- Alpers, B. (2012). The mathematical reasoning competency for a practice-oriented study course in mechanical engineering. Proc. 40th SEFI Ann. Conf., Brussels: SEFI.
- Alpers, B. et al. (2013). A framework for mathematics curricula in engineering education. Brussels: SEFI.
- Gross, D. et al. (2013). Engineering Mechanics 1. Statics. 2. Ed., Dordrecht: Springer.
- Gross, D. et al. (2016). Technische Mechanik 1. Statik. 13. Auflage, Berlin-Heidelberg: Springer.

# Support for Teaching Mathematics online

Mgr. Martina Bobalová, Ph.D

## Abstract

Mathematics plays an important part in the study programmes of technical and economic universities. In cases where mathematics is taught to students as a subsidiary subject, the substantially limited number of hours is an issue as well. This results in the restriction or complete omission of certain topics and there is usually no time at all left for the introduction to applications of mathematical methods in economics. According to study results, mathematical subjects rank among the more difficult ones and the number of students who need to retake or completely fail the course has been increasing.

These days, there is an ever growing amount of materials available to the students online. Therefore finding various sources to support teaching is not a problem, while selecting suitable sources for particular studies may prove demanding.

A website has been created for support of Mathematics 1. The teaching programme is designed as a single page JavaScript application built over a modern Vue.js framework with UI component Vuetify plugin. Thanks to the use of this framework, the application is responsive and may be used on mobile devices as well as desktop PCs. MathJax, a JavaScript library, is used for displaying mathematical expressions and equations in the web browser, which renders mathematical expressions in LaTeX dynamically and quickly. Graphs are also depicted dynamically, using the MathJS and Plotly JavaScript libraries. MathJS for programme calculation of the entered graph functions and Plotly for displaying. Dynamic display of mathematical expressions and graphs brings the benefit of excellent text and graph legibility with various DPI. The website can be found on the faculty server <https://vyuka.fbm.vutbr.cz/matematika1/> and the link is given to students in e-learning.

The website content includes problems and their results, grouped in accordance with the subject syllabus. There are several model problems for each chapter, including the display of the individual calculation steps and potential comments on the process and result. Topics crucial for follow-up studies are emphasized. There is a sufficient number of problems for practice of the subject matter for each topic, so that the student can prepare for tests in the seminars and final exams. Tests for self-check and review of the respective knowledge are available as well. The website allows students to practice various problems from the individual subject chapters in a targeted manner. In the course of the semester, the tutor can add problems based on the students' current needs and the students have all the subject materials in one place.

## Mathematics in a programme for Electric Systems Design and Innovation

Torstein Bolstad<sup>1</sup>, Lars Lundheim<sup>1</sup>, Morten Nome<sup>2</sup> and Frode Rønning<sup>2</sup>

<sup>1</sup> Department of Electronic Systems, NTNU, Trondheim, Norway

<sup>2</sup> Department of Mathematical Sciences, NTNU, Trondheim, Norway

Engineering education has from early on experienced a tension between theory and practice, between academic and professional aims, and recent studies show that this tension still persists (Carvalho & Oliveira, 2018). Part of this tension concerns the role of mathematics in engineering, and the connection and perceived relevance of mathematics (Flegg et al., 2012; Gueudet & Quéré, 2018). Many models exist for organising the teaching of mathematics for engineers, from providing mathematics in general courses to a large variety of different study programmes, to a model where mathematics courses are specially designed for particular engineering programmes.

Currently, the mathematics courses at NTNU are not programme specific, but recently a project has been initiated with a revised scheme in mathematics, so far only for the programme Electronic Systems Design and Innovation (ELSYS). This project entails a change in the order of the mathematical topics, as well as a shift in the emphasis of the various topics. One goal of the project is for students to experience a stronger link between mathematics and engineering, thereby experiencing greater relevance and mastery of the mathematics they are exposed to. Another, more long term goal, is to develop a more programme driven approach, inspired by the CDIO initiative (Crawley et al., 2014). This means that mathematics should be an essential part of the students' formation, a way of working and thinking, and not just a set of tools to be acquired or topics to be learned (Scanlan, 1985).

The revised scheme's benefits are twofold: On the one hand, the scheme makes it possible to apply and mathematically justify principles in electronics, taught in courses in parallel with the mathematics course. This serves as a motivation for abstract mathematical topics for otherwise less motivated students. On the other hand, teachers of engineering topics find that they can more easily build on recently taught principles from mathematics and thereby help the students to acquire a deeper understanding of concepts in electrical engineering. One example is that by actively using results from linear algebra, a well-founded explanation of the superposition principle in circuit theory can be given. Another example is how awareness of how and when 2nd order differential equations are taught in mathematics can give a solid basis for the teaching of oscillators in the second semester of the ELSYS programme.

We will report from the first year of implementation of the revised scheme and present explicit examples of connections between mathematics and electronic systems and show how the two fields can influence, interact and support each other. Experiences from teachers and preliminary results from student surveys will also be included.

### References

- Carvalho, P., & Oliveira, P. (2018). Mathematics or mathematics for engineering? In *Proceedings from 2018 3rd International Conference of the Portuguese Society for Engineering Education (CISPEE)*.
- Crawley, E. F., Malmqvist, J., Östlund, S., Brodeur, D. R., & Edström, K. (2014). *Rethinking engineering education. The CDIO approach* (2nd Ed.). Cham: Springer.
- Flegg, J., Mallet, D., & Lupton, M. (2012). Students' perceptions of the relevance of mathematics in engineering. *International Journal of Mathematics Education in Science and Technology*, 43(6), 717–732.
- Gueudet, G., & Quéré, P.-V. (2018). "Making connections" in the mathematics courses for engineers: The example of online resources for trigonometry. In V. Durand-Guerrier, R. Hochmuth, S. Goodchild, & N. M. Hogstad (Eds.), *Proceedings of INDRUM 2018. Second conference of the International Network for Didactic Research in University Mathematics* (pp. 135–144). Kristiansand, Norway: University of Agder and INDRUM.
- Scanlan, J. O. (1985). The role of mathematics in engineering education: An engineer's view. *International Journal of Mathematical Education in Science and Technology*, 16(3), 445-451.

## Integrating Mathematics and Engineering Education at TU Delft:

### meet PRIME<sup>1</sup> pre, during and post COVID-19

Annoesjka J. Cabo<sup>2</sup>

At the Delft University of Technology (TU Delft) it has been decided that service mathematics for non-mathematics students is taught by mathematicians. Yearly about 20,000 students from eight different faculties and 20 programmes are served by 48 full time equivalent (fte) mathematics lecturers. To address the well-known challenges of teaching service mathematics for non-specialists, a large scale innovation programme has been launched: PRogramme of Innovation in Mathematics Education ([PRIME](#)). The goals of PRIME are 1) to activate students, 2) to increase efficiency of learning for students, 3) to increase efficiency of teaching for lecturers and 4) to improve transfer of mathematics to engineering.

The programme has been quite successful: a new, activating learning method has been developed as well as open educational material that is widely used and shared; a research programme has started around PRIME; at TU Delft, students asked for a programme in PRIME format for their mechanics courses.

Achieving transfer of mathematics to engineering is still the biggest challenge. Incorporating examples from specific engineering disciplines in the mathematics courses turned out to be unsuccessful. Currently, context videos are being developed in collaboration with professors from the engineering faculties. This seems to be a promising way forward.

The aims of this presentation are firstly to present PRIME to the audience, to share lessons learned and to collect feedback and ideas for future developments. Secondly, the teaching method turned out to be COVID-19 proof, which gives rise to the question how to continue after the pandemic. To conclude, a discussion on how to assess the mathematical competencies achieved in PRIME would be very interesting, since this is one of PRIME's current prime focus of research.

---

<sup>1</sup> PRogramme of Innovation in Mathematics Education

<sup>2</sup> Delft University of Technology, Faculty EEMCS, Delft Institute of Applied Mathematics;  
Mekelweg 4; 2628 CD Delft; the Netherlands

# Mathematics for engineers: a case study about assessing knowledge and competencies

Cristina M.R. Caridade <sup>(1)</sup>, Deolinda M. L. D. Rasteiro <sup>(1)</sup>, Daniela Richtarikova <sup>(2)</sup>

[caridade@isec.pt](mailto:caridade@isec.pt); [dml@isec.pt](mailto:dml@isec.pt); [daniela.richtarikova@stuba.sk](mailto:daniela.richtarikova@stuba.sk)

<sup>(1)</sup> Coimbra Institute of Engineering, Portugal

<sup>(2)</sup> Slovak University of Technology in Bratislava

**Abstract:** Evaluating is a process that involves conceptions, values, principles, theories, goals, desires, trajectories, and when the process focuses on education it becomes potentially more complex, creating challenges to its verification and recording [1]. Evaluating in education should be understood, not only as a product of education and classification of students, courses, institutions, but mainly as a process with educational, pedagogical, and psychological characteristics. Educational assessment, however, consists of much more than randomly designed tests and tests designed to verify that the student has “learned the content of Curricular Unit”. It is a complex process that, when well planned and executed, can become a powerful tool in building school learning.

The educational assessment process presents an interdependence between knowledge learning and competencies development [2, 3]. In this perspective, knowledge and competencies are processes that are articulated, but not confused. In fact, the use of knowledge is a strong requirement in the process of building competencies. However, the more human actions that require the deepening or organization of knowledge, the more time is required for the development of competencies. Thus, one of the major future challenges facing evaluation processes is to privilege, among the aspects to be evaluated, the development of competencies [4, 5]. According to B. Alpers et al. in the Framework for Mathematical Curricula in Engineering Education [6], previously proposed in the Danish KOM project [7] the mathematical competencies are: (C1) thinking mathematically; (C2) reasoning mathematically; (C3) posing and solving mathematical problems; (C4) modelling mathematically; (C5) representing mathematical entities; (C6) handling mathematical symbols and formalism; (C7) communicating in, with, and about mathematics and (C8) make use of aids and tools for mathematical activity. Thinking about this, it was proposed ‘New Rules for assessing Mathematical Competencies’ project to change the educational paradigm and to get a common European teaching and learning system based on mathematical competencies rather than contents [<https://rules-math.com/>].

The RULES\_MATH project working groups have developed a set of “Guide for a Problem” in the different areas of mathematics that are intended to provide some examples of proposed forms of assessment and competence-based activities. The materials are available to all project partners to apply to different students from different courses and institutions. One such material is AC3 which aims to evaluate students about complex numbers.

The assessment described in AC3 was implemented with linear algebra students, since it is in this course that a small review of the complex numbers is made, and the complex numbers in the Euler's form is presented to students for the first time. The study was carried out in a group of 126 students of Linear Algebra from Biomedical, Electromechanical and Mechanical Engineering in Coimbra Institute of Engineering. The results obtained were satisfactory since at almost questions (7 out of 9) students were able to obtain a positive grade. The performed assessment test also permits to identify level differences between the courses where it was applied. Relating the test itself as a tool to assess competencies, we may conclude that it covers most of the competencies that we need to evaluate, and its difficulty is adequate to our students.

During the world pandemic of Covid 19, the need to transform teaching and learning methods was urgent and their impact on evaluation was also felt. As a result, this paper reflects the necessary adjustments on assessment that were proposed to students always having in mind the outcomes of Rules\_Math project.

**Keywords:** Assessment, Significant Learning, Competencies, Mathematics, Engineering

**Mathematics Subject Classification:** Primary 97F50, Secondary 97B40, 97B50, 97C40, 97C50

## References:

- [1] Ross, T. Exploring mathematical competencies, 2010, 24:5. Available at: <https://research.acer.edu.au/resdev/vol24/iss24/5>
- [2] Caridade C.M.R., Rasteiro D.M.L.D. Evaluate Mathematical Competencies in Engineering Using Video-Lessons. In: Martínez Álvarez F., Troncoso Lora A., Sáez Muñoz J., Quintián H., Corchado E. (eds) International Joint Conference: 12th International Conference on Computational Intelligence in Security for Information Systems (CISIS 2019) and 10th International Conference on European Transnational Education (ICEUTE 2019). CISIS 2019, ICEUTE 2019. Advances in Intelligent Systems and Computing, vol 951. Springer, Cham
- [3] Boesen, J., Lithner, J., Palm, T. Assessing mathematical competencies: an analysis of Swedish national mathematics tests, *Scandinavian Journal of Educational Research*, 2018, 62:1, 109-124, DOI: 10.1080/00313831.2016.1212256
- [4] Niss, M., Høgaard, T. (eds.). *Competencies and Mathematical Learning. Ideas and inspiration for the development of mathematics teaching and learning in Denmark*, English Edition, Roskilde University, 201.
- [5] Niss, M., Bruder, R., Planas, N., Turner, R., Violla-Ochoa, J.A. (2017) Conceptualisation of the Role of Competencies, Knowing and Knowledge in Mathematics Education Research. In: Kaiser G. (eds) *Proceedings of the 13<sup>th</sup> International Congress on Mathematical Education. ICME-13 Monographs*. Springer, Cham.
- [6] Alpers, B. et al, *A framework for mathematics curricula in engineering education*, SEFI, 2013. Available online at: <http://sefi.htw-aalen.de/>
- [7] Niss, M., *Mathematical Competencies and the Learning of Mathematics: The Danish KOM Project*, In *Proceedings of the 3rd Mediterranean*, 2003.
- [8] RULES\_MATH project. <https://rules-math.com/>

# Design principles for digitally assessing linear algebra

Tracy Craig and Alisa Lochner  
University of Twente, the Netherlands

## **Abstract:**

Digital testing is an increasingly popular option for assessment of large classes in many subjects including mathematics. Such testing takes on a variety of forms including automated assessment of multiple choice questions (MCQ) and final answer items. Criticism of MCQ and final answer assessment can be justifiable, and designing items that are well suited to such assessment is challenging. At the University of Twente we have practical experience with digital testing in a few of our first-year service mathematics courses such as calculus and linear algebra. After several cycles of analysis of digital testing in linear algebra we have refined a set of design principles to inform test instrument design. In this presentation we shall describe the process of refinement and will present and argue for these design principles when pursuing digital testing assessment of MCQ and final answer items in Mathematics.

**Keywords:** digital testing; service teaching of mathematics; linear algebra; design principles

**Theme:** “How to assess competencies” is the best fit, although it is within the context of service courses, that is teaching linear algebra to physics and engineering students.

Topic: Subjects related to teaching adequation to COVID-19 (main)

## **Experience gained during online teaching**

Marie Demlova, Petr Habala

Department of Mathematics, Faculty of Electrical Engineering, Czech Technical University in Prague

Contact: [mail.demlova@fel.cvut.cz](mailto:mail.demlova@fel.cvut.cz)

Recent transfer to distance education forced many teachers to reshape their courses and reach for alternative ways to interact with students and transmit knowledge. Typically, the tools that we use have been around for quite a while, but they have not been used under normal conditions for various reasons.

However, now that we have about two semesters worth of experience, it is useful to look at these tools with new eyes. Obviously, the utility of tools depends on circumstances, and given how much the circumstances changed, it is not surprising that the mutual balance of advantages and disadvantages of various approaches may come up different.

Perhaps even more interesting is another question. We were, in a way, forced to adopt tools that were previously not found useful, sometimes even though dangerous or unworthy. Having to face these tools now, is there a reason to perhaps correct some of our appraisals? Generally speaking, it seems that our base understanding does not need corrections, but there are some aspects that do deserve a new look, and they can have an impact on general acceptance of some tools.

In this paper, we will look at the three major components of a typical course: a lecture, a practical class, and a homework. We will look at some typical (in our experience) adaptations to new circumstances. Typically, these approaches are not recent, and we will recall expectations (perceived pros and cons) that seemed common among our colleagues. Then we discuss our new perceptions and experiences.

Given that we all hope that university education will again return to on-site learning, perhaps the most intriguing question is whether our recent experiences will have some impact on the future. Of course, we cannot say with any certainty, but it does seem that some of the tools that we were forced to employ did prove their worth, and it remains to be seen whether we succeed in integrating them to our traditional setup.



# COVID-19's Impact on the Quality of Educational Process and the Academic Performance as Viewed by IT Students: A Case Study in Text Mining

Natalja Maksimova, Avar Pentel and Olga Dunajeva

Virumaa College of Tallinn University of Technology, Järveküla tee 75, 30322 Kohtla-Järve, Estonia

[natalja.maksimova@taltech.ee](mailto:natalja.maksimova@taltech.ee)

[avar.pentel@taltech.ee](mailto:avar.pentel@taltech.ee)

[olga.dunajeva@taltech.ee](mailto:olga.dunajeva@taltech.ee)

## ABSTRACT

The COVID-19 pandemic brought changes to the educational process worldwide. In an effort to contain the spread of the virus, universities and colleges around the world have been forced to suspend face-to-face or classroom learning and move to distance e-learning. Estonia's success in the digital revolution extends to higher education. When during the first wave of the coronavirus pandemic in March 2020 the country was forced to lockdown, Estonia's universities and colleges switched to remote teaching in just one day, because digital learning platforms and materials were already in use. Thanks to digitalisation at most higher education institutions dropout rates did not increase during this spring term. However, the results of surveys conducted at many universities highlight such global negative aspects as an increased workload, problems with practical works, social isolation, lack of direct communication, lack of learning motivation, mental and emotional issues. In order to get more detailed students' feedback about their studies during the lockdown such survey method as students essay analysis can be applied. Text mining and sentiment analysis techniques proposed in literature can be effectively used to analyse the students' sentiments and views on the quality of educational process with the aim to improve online learning and students' motivation and emotional state.

In this paper we describe the results of the text mining and sentiment analysis case study with the aim to analyse the COVID-19's impact on the quality of educational process and the academic performance of IT students of Virumaa College of Tallinn University of Technology through an essay assignment. In this study we analyse and compare students' feedback about their studies in the lockdown during the first and the second wave of the COVID-19 pandemic. We also provide the methodology of the analysis, find influencing factors and develop recommendations for improving online learning and students' motivation and emotional state.

In order to examine students' opinions, the following essay questions were formulated: (1) "How has the first wave of the COVID-19 changed studying in Virumaa College? Please, discuss online learning platforms, tools, methods, changes in assessment." (2) "How has the first wave of the COVID-19 changed your academic performance? Please, discuss the advantages and disadvantages of online learning." (3) "How has quality of the educational process changed in Virumaa College after the first wave? Please, discuss the study process during the second wave of COVID-19 and the lessons learned from the first wave." (4) "How has the second wave of the COVID-19 influence your academic performance? Please discuss your problems associated with online learning and suggest some ways to improve online education in the Virumaa College." To identify students' sentiments and determine the potential influencing factors descriptive textual analytics, sentiment analysis techniques and textual data visualizations are used in this study.

As a result of this study students' sentiments and opinions about the quality of educational process and their academic performance during the first and the second wave of the COVID-19 pandemic were compared and the most influential factors were determined using textual analytics and sentiment analysis techniques. Based on this study the recommendations were developed to improve online learning and students' motivation and emotional state. The methodology of the analysis provided in this paper can be helpful for further using as well as implemented by any higher education institution.

The results of this study coincide with the results of other studies conducted by the universities of Europe and America. But the provided methodology of textual analytics is helpful in any particular case and can be used in following researches to derive useful recommendations.

## KEYWORDS

Academic performance, COVID-19, sentiment analysis, students' essay, text mining.

# On embedding dynamic mathematical tools into computer-aided assessment systems - preliminary findings from a pilot study

Mats Brunström, Maria Fahlgren, Mirela Vinerean, and Yosief Wondmagegne

Department of Mathematics and Computer Science, Karlstad University

At Karlstad University, a developmental project to increase first year engineering students' learning in mathematics was initiated in 2015 (Mossberg, Vinerean-Bernhoff & Brunström, 2016), based on experiences from research projects at upper secondary school (e.g. Brunström & Fahlgren, 2015). The focus has been on the development of student activities designed for a dynamic mathematics software (DMS) environment, in this case GeoGebra. The intention behind these activities is to deepen students' understanding by providing learning environments where they can explore and communicate mathematics with peers. Course evaluations indicate that students appreciate this part of the course. Since the project turned out well, today these activities constitute mandatory parts of the first year engineering mathematics courses at Karlstad University. However, due to the limited time available for the course teachers, the feedback provided to students on their submitted answers has often been delayed since it has been a challenge for the teacher to assess (in a short time) a large number of student responses. One way to reduce the time-consuming correction work is to outsource it by using technology (Rønning, 2017).

Nowadays, there are technologies, often denoted as computer-aided assessment (CAA) systems, which automatically provide students immediate feedback on their submitted answers. However, these systems have so far mainly been used for assessing basic mathematical procedural skills, most often by categorizing final answers as being right or wrong. On the other hand, it is possible to embed DMS environments, such as GeoGebra into a CAA system (Sangwin, 2015). Accordingly, we found it interesting to investigate how a carefully designed combination of these two specific types of technology could be utilized to increase first year engineering students' engagement and conceptual understanding.

In the autumn of 2020 we conducted a pilot study in a calculus course for first year engineering students. The focus of the study was to find out the feasibility of the interplay between these digital environments in designing different types of learning tasks as well as to get a deeper understanding on those prevailing student strategies when performing these tasks. Refined procedures from the pilot study as well as useful insights from the preliminary results of the study help us develop suitable forms of feedback based on various student responses. How this contributes to enhance first year engineering students' engagement in and conceptual understanding of mathematical contents is in turn to be explored in an upcoming research project. In this presentation, we will share our experiences as well as findings from the pilot study.

## References

- Brunström, M., & Fahlgren, M. (2015). Designing prediction tasks in a mathematics software environment. *International Journal for Technology in Mathematics Education*, 22(1), 3-18. doi:10.1564/tme\_v22.1.01
- Mossberg, E., Vinerean-Bernhoff, M., & Brunström, M. (2016) An exploratory approach to engineering mathematics using GeoGebra. Poster presented at *the 18<sup>th</sup> SEFI Mathematics Working Group Seminar, June 27-29*. Gothenburg, Sweden
- Rønning, F. (2017). Influence of computer-aided assessment on ways of working with mathematics. *Teaching Mathematics and its Applications*, 36(2), 94-107.
- Sangwin, C. (2015). Computer aided assessment of mathematics using STACK. In S. J. Cho (Ed.), *Selected regular lectures from the 12th international congress on mathematical education* (pp. 695-713). Switzerland: Springer.



## Students as partners in the development of math support center.

Ane Sofie Andersen, Even Vehus, Filmon Berhe Mebrahtom, Jenny Johannessen, Silje Hatlevik, Teklematiam Weldehawariat, Rolkana Alo, Benjamin Ims, Ali Shahab Rezaii, Preben Forsland, Lillian Egeland, Eva Dønnestad and Thomas Gjesteland

University of Agder

University of Agder (UiA) have two math support centers, one at each campus. The support centers are called MatRIC Drop-in. Drop-in is part of MatRIC, a nationally center for excellent in education. At Campus Grimstad the main users of Drop-in are engineering students in their bachelor's program. The Drop-in is employed with senior students. We have experienced that too few first-year students are aware of the support centers. We have also seen that many students at-risk for failing mathematics courses do not use Drop In. This is also supported in previous studies e.g. Pett and Croft (2008) and Mac an Bhaird et. al., (2009). The review of Matthews et. al., (2013) also address non-users of support centers. Studies suggests fear of showing lack of knowledge could be a reason why students do not use math support centers. To address this problem, Drop-in have included the students as partners in developing Drop-in as a dynamic meeting place for students. The focus is human connection meeting between students with need for help and the senior students working at Drop-in. We have increased the human connection competence for the students working at Drop-in. The goal is to increase the student engagement and lower the threshold for students to contact, and use Drop-in. In this paper we present the measures, developed in partnership with students, to increase Drop-in participations. The students have branded MatRIC Drop-in and actively used social media to reach out to students.

Godfrey Pell, Tony Croft, (2008) Mathematics support—support for all?, *Teaching Mathematics and its Applications: An International Journal of the IMA*, Volume 27, Issue 4, December 2008, Pages 167–173, <https://doi.org/10.1093/teamat/hrn015>

Ciarán Mac an Bhaird, Tadhg Morgan, Ann O'Shea (2009), The impact of the mathematics support centre on the grades of first year students at the National University of Ireland Maynooth, *Teaching Mathematics and its Applications: An International Journal of the IMA*, Volume 28, Issue 3, September 2009, Pages 117–122, <https://doi.org/10.1093/teamat/hrp014>

Janette Matthews, Tony Croft, Duncan Lawson, Dagmar Waller, (2013) Evaluation of mathematics support centres: a literature review, *Teaching Mathematics and its Applications: An International Journal of the IMA*, Volume 32, Issue 4, December 2013, Pages 173–190, <https://doi.org/10.1093/teamat/hrt013>

## **Improving engineering students' engagement in calculus tasks: Contributions of an oral assessment in group work.**

Thomas Gjesteland<sup>1</sup>, Vegard Lima<sup>1</sup>, Yusuf F. Zakariya<sup>2</sup>, Hans Kristian Nilsen<sup>2</sup>

<sup>1</sup>Department of Engineering Sciences Faculty of Engineering and Science, University of Agder

<sup>2</sup>Department of Mathematical Sciences, Faculty of Engineering and Science, University of Agder

In our engineering program, the first semester mathematics class suffers from high failure rate. Our first semester mathematics course consists of basic calculus with 7.5 ECTS points. It has four hours of lecture per week and two hours of problem solving sessions. The students have to submit and pass 4 out of 5 mandatory assignments in order to take the final exam. The grading is based on final exam alone. We experienced that the attendance in the problem solving sessions was very low. To increase the attendance at these sessions, we included optional oral assessment of the mandatory assignments. If the students were present during the problem solving hours and worked through the problems with their study group, we would ask the group to give an oral presentation of their work. The students would then get the assignment approved. The students who did not attend the sessions had to deliver the assignments on paper. This initiative increased the attendance at the problem solving session significantly. We will present our experience by using the optional oral assessment.

## **Engagement of engineering students with maths support**

Farhana Gokhool

### Abstract

Many engineering students arrive at university underprepared for the mathematical demands of their courses. Several higher education institutions in the UK, Ireland, Germany and the Czech Republic have introduced mathematics support as a way of assisting these students. One of the aims of maths support is to reduce the high failure/dropout rates of engineering students. In most institutions, maths support is offered to students on a voluntary basis – they can choose to engage with it or not. Typically, Engineering students make up the majority of those who ask for support; despite this, there are still a large number of engineering students (whose results indicate that they would benefit from assistance) who do not engage with maths support.

In this presentation, we will briefly outline the nature of maths support and how it typically operates, reviewing its extent across Europe. Then we will give some results from Coventry University showing the high volume of engagement with maths support by engineering students. We will then go on to explore how their engagement with maths support differs by a range of demographic factors including ethnicity, gender, age, course, disability status, year of study etc.

## **New Guidelines for the National Curriculum Regulations for Engineering Education in Norway.**

Mette Mo Jakobsen<sup>1,2</sup>, Inger Johanne Lurås<sup>3</sup>, Arvid Siqveland<sup>4</sup>, Anders Tranberg<sup>5</sup>, Thomas Gjesteland<sup>2</sup>

<sup>1</sup>Universities Norway (UHR)

<sup>2</sup>Department of Engineering Sciences, Faculty of Engineering and Science, University of Agder

<sup>3</sup>Department of Education and Quality in Learning, University of South-Eastern Norway

<sup>4</sup>Faculty of Technology, Natural Sciences and Maritime Sciences, University of South-Eastern Norway.

<sup>5</sup>Faculty of Science and Technology, University of Stavanger

The Norwegian bachelor of engineering education is based on national curriculum regulations given by the Norwegian Ministry of Education and Research. Revised regulations were adopted on 18<sup>th</sup> of May 2018. Universities Norway (UHR) was given the task of developing and approve guidelines for these regulations. The curriculum regulations and the guidelines are based on the Norwegian qualifications framework for higher education. Guidelines are given on both knowledge, skills and general competencies. UHR and Centre for Research, Innovation and Coordination of Mathematics Teaching (MatRIC) organized a workshop, with representatives from all institutions with engineering programs, to work on the update of the guidelines for mathematics. The new guidelines also include developments in digital competence, basic programming and social sciences.

The guidelines refer to the SEFI report “A Framework for Mathematics Curricula in Engineering Education”, as a benchmark for content-related competencies, knowledge, and skills. An overview of the new guidelines for mathematics in the Norwegian engineering education will be presented.

## **Measures for refreshment of mathematical competencies and calculation skills at beginning of a degree**

Karin Landenfeld, Jonas Priebe, Regina Abraham

*Hamburg University of Applied Sciences, Germany*

### **Abstract**

Lack of previous mathematical knowledge as well as inadequate calculation and solution skills are one of the main hurdles for students when starting a degree. Various measures have been tried out at universities for years to refresh this prior knowledge before or during the first semester.

In this contribution we would like to present different approaches at our university to improve the start of studies, especially to develop the mathematical skills using an online learning environment. This includes pre-mathematical courses in blended learning format in presence and online, an extended course of study with supplementary specially designed mathematic lectures and tutorials, as well as a refresher course during the first semester.

We will describe the measures and present the results of math knowledge tests before and at the end of the measure. In addition, we would like to present the concept and the results of the mathematics preliminary courses, which have taken place entirely online for the last two semesters.

---



## Active learning in mathematics – what is it good for?

Duncan Lawson, Coventry University

For many years there has been growing dissatisfaction with the dominance of the traditional lecture within the learning experience of students studying mathematics in higher education. A range of alternatives or complements to the traditional lecture has been explored in different settings and with different groups of students. Approaches such as problem-based learning, inquiry-based learning, flipped classroom, etc. are all recorded in the literature. Such approaches come under the banner heading of active learning.

Following what has been called a “landmark study” of active learning in STEM by Freeman et al (2014), the case for incorporating elements of active learning in the student experience has become almost overwhelming. Indeed, the Conference Board of the Mathematical Sciences in the US (a grouping of leading mathematical organisations such as MAA, AMS, SIAM, etc) issued a position statement (CBMS, 2016) which stated:

*We call on institutions of higher education, mathematics departments and mathematics faculty, public policy makers and funding agencies to invest time and resources to ensure that effective active learning is incorporated into post-secondary mathematics classrooms*

In this presentation, we will briefly review the evidence presented by Freeman et al (2014). Following this, we will explore ways in which active learning can be incorporated in higher education, even in the large classes that are typical of engineering mathematics courses. This will include consideration of one of the common objections to active learning that it takes longer and therefore less material can be covered.

Finally, we will review the mathematical competencies for engineers set out in the SEFI curriculum document (Alpers et al, 2013) and consider whether active learning or traditional lecturing is more likely to develop these competencies in students.

### References

Alpers, B (2013) *A Framework for Mathematics Curricula in Engineering Education*, SEFI, Brussels.

CMS (2016) *Active learning in post-secondary mathematics education – position statement*, Conference Board of the Mathematical Sciences.

Freeman, S, Eddy, S, McDonough, M, Smith, M, Okoroafor, N, Jordt, H and Wenderoth, M (2014) Active learning increases student performance in science, engineering and mathematics, *PNAS*, 111(23):8410-8415.

## Abstract

Keywords: Digital notebook, OneNote Class NoteBook, Covid-19, supervising, digital didactics

## New use of digital teaching tools to connect with students during Covid-19

### Problem:

Due to Covid-19 the NTNU Campus shut down in March 2020. This led to a need of new teaching tools to connect with students digitally. How could we teach mathematics when we are not at the same location as the students?

### Method:

- Teaching and supervising in OneNote Class NoteBook by screen sharing and digital handwriting. (Fig.1)
- In problem solving students are sharing their own notes in OneNote Class NoteBook with their teacher or teaching assistant. (Fig. 2)
- Two-way conversation with individual or groups of students.

### Results:

Bridging the gap of the traditional and digital teaching situation by using OneNote Class NoteBook for both teaching and supervising.

Teaching all students by sharing OneNote Class NoteBook	Supervising a student by sharing his notes in OneNote Class NoteBook								
<p><b>Fig.1</b></p> <p>e.k.s. Løyping av ei bølgefremning med start og randbetingelser/krav</p> $4u_{xx} = u_{tt}$ <p><math>u(0,t) = 0</math> <math>u(\pi,t) = 0</math> <math>u(x,0) = \sin x</math> <math>u_t(x,0) = 0</math></p> <p>Finsteds løsning p.g.m. 4 krav</p> <p><math>0 \leq x \leq \pi, 0 \leq t &lt; \infty</math></p> <p>Swingende streng - tallet 4 er knyttet til tykk streng og kor masse osv. er utpant.</p> <p>Sjekk: Stemmer det at <math>u(x,t) = \sin(x) \cos(2t)</math> og i oppfylle start og randbetingelser?</p> <p><math>u(0,t) = \sin 0 \cdot \cos(2t) = 0 \cdot \cos 2t = 0</math> ok! <math>u(\pi,t) = \sin(\pi) \cdot \cos 2t = 0 \cdot \cos 2t = 0</math> ok! <math>u(x,0) = \sin x \cdot \cos(2 \cdot 0) = \sin x</math> ok! <math>u_t(x,t) = -\sin(x) \cdot \sin(2t) \cdot 2 = -2 \sin(x) \sin(2t)</math> <math>u_t(x,0) = -2 \sin(x) \cdot \sin(0) = 0</math> ok!</p> <p><math>u(x,t) = \sin(x) \cos(2t)</math> er enstydig løsning på korleis strengen beveger seg.</p>	<p><b>Fig.2</b></p> <p>4.1 Rette linjer, 4.2 Grafregning på lommerregner, 4.3 Grafisk avlesning</p> <p><math>x - 2 = y - 2 + 1</math></p> <p><math>a = \frac{y}{x} = 2</math></p> <p><math>a = \frac{y}{x} = 2</math></p> <p>Tabell</p> <table border="1"><tr><td>x</td><td>-2</td><td>-1</td><td>0</td></tr><tr><td>y</td><td>-3</td><td>-1</td><td>1</td></tr></table> <p><math>x - 2 = y - 2 + 1</math> <math>y = 2(-2) + 1</math> <math>y = -4 + 1 = -3</math> <math>x = -1</math> <math>y = 2(-1) + 1</math> <math>y = -2 + 1 = -1</math></p>	x	-2	-1	0	y	-3	-1	1
x	-2	-1	0						
y	-3	-1	1						

Arnhild Lunde, Assistant Professor, NTNU

Tonje Vedde Fiskerstrand, Assistant Professor, NTNU

Dear organizers,

Please find an abstract for a presentation below. The topic is: "How to assess competencies", but it could also fall under the category: "Subjects related to teaching adequation to COVID-19". It is a bit of both.

Authors:

Vegard Lima, Øystein Midttun, Sverre Lunøe-Nielsen

Affiliation:

University of Agder

Title:

Alternative mathematical problems for digital home exams

Abstract:

Since the COVID-19 pandemic started in March 2020, student evaluation in mathematics changed to digital home exams with all notes and computational tools available. This change has been very problematic as the traditional method to test the students' abilities in mathematics is to present an equation and ask the students to calculate the answer. With online tools available, complete solutions to these problems are just a few clicks away. But what we really want to test is the students understanding of mathematics, not their ability to use Symbolab, Photo Math, Wolfram Alpha, or equivalent tools.

This project focuses on alternative methods of presenting mathematical problems such that the students need to think for themselves instead of using online tools. We will test these new methods as part of a test exam in a mathematics course at the University of Agder during the spring 2021.

Best regards,

Øystein Midttun

**SimReal – Visualization and Simulation in Mathematics/Physics/Statistics/ICT**

SimReal is a digital tool in mathematics, physics, statistics and ICT that is under construction at the University of Agder. The main idea of SimReal is, through interactive visualizations and simulations, to give the students a deeper understanding of these subjects.

After many valuable inputs from the students the last two years, SimReal is now a more ‘complete’ learning package that can be used in teaching and learning. In addition to visualizations and simulations, SimReal now contains videos, streaming, exercises with solutions, modeling, programming (SimReal, JSXGraph, JavaScript, Jupyter, Python, Matlab) and applications.

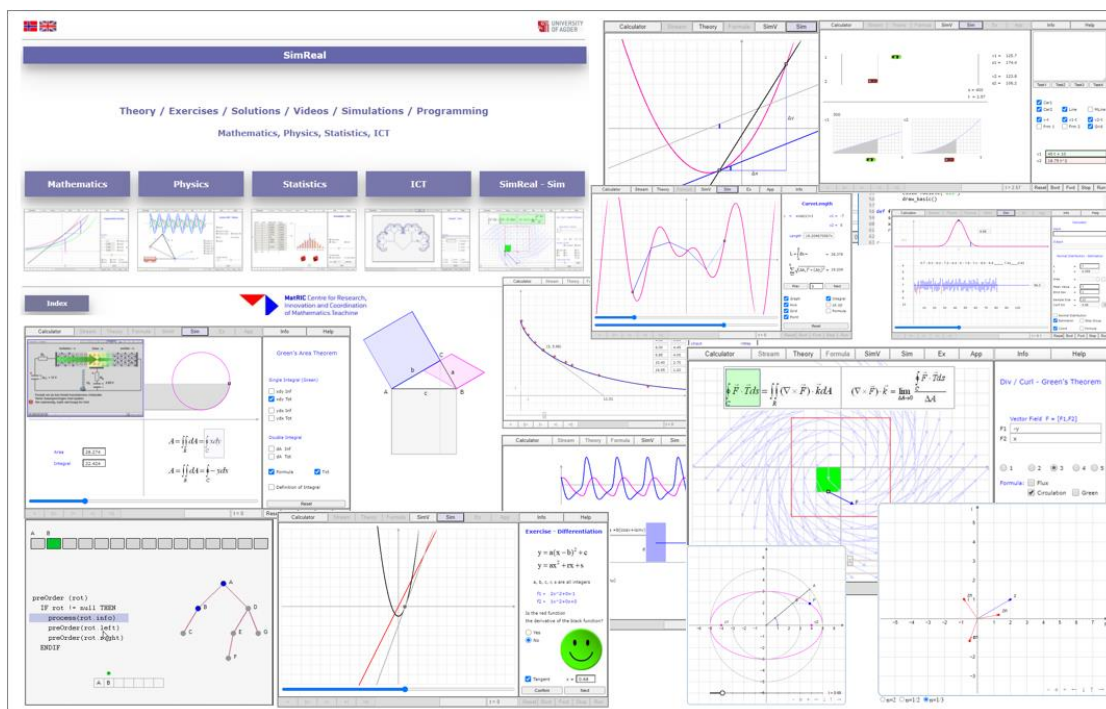
SimReal is programmed in an extended version of JavaScript and therefore executable in web browsers on every kind of devices, including mobile phones.

SimReal contains a complete and open programming library and programming code. Programming can be done by students and teachers using Google Chrome.

SimReal is modularized so that every part can be accessed directly.

Parts of the exercises are self-generated with feedback. This way there will be an unlimited number of exercises. A connection between SimReal and the interactive geometry-, plotting- and visualization-tool JSXGraph is established. Later there will also be a connection between SimReal and STACK, the world-leading open-source online assessment system for mathematics.

Students give very positive feedback of SimReal, and exams show that the students add more detailed explanations to their calculations.



In the picture there are some examples from parts of SimReal: Main menu, differentiation, curve length, triple integral, probability, estimation, semiconductors, Pythagoras, Green’s Theorem, Green’s Area Theorem, recursive programming, self-generated exercises, ellipse, complex roots and mathematical modeling (complex numbers and differential equations in the descriptions of the movement of a piston and a boat and coupled differential equations in pandemic- and cancer-studies, programmed in SimReal, JSXGraph and Python.

# Students' self-awareness of learning agility: a case-study

**J. Mendonça<sup>1</sup>, C. M. A. Pinto<sup>2</sup>, L. Babo<sup>3</sup>**

<sup>1</sup>School of Engineering, Polytechnic of Porto  
Porto, Portugal

<sup>2</sup>Centre for Mathematics, University of Porto  
Porto, Portugal

<sup>3</sup>Porto Accounting and Business School, Polytechnic of Porto  
CEOS.PP, Porto, Portugal

**Keywords:** Mental Agility, People Agility, Learning Agility, Engineering Students, Math Courses, Self-Awareness

## ABSTRACT

The Education system has suffered major changes, particularly since the 4th Industrial Revolution. The market place is characterized by volatility, uncertainty, complexity, and ambiguity, also known as VUCA. The demands from these VUCA market put high emphasis on Higher Education Institutions, with respect to their courses' offers and performance of their future graduates. HEIs need to promote new Education models to foster higher employability and adequacy of their graduates to the challenges ahead. Future graduates are expected to have high critical-thinking and communication skills, be eximious collaborators and problem-solvers, and be motivated and resilient to face changes. These are some of the traits of people with high learning agility, which is a key factor in the VUCA world. Learning agility is the ability and willingness to apply in new or different situations the knowledge acquired from experiential learning. Learning agility can be measured using five dimensions: mental agility, people agility, change agility, results agility and self-awareness.

In this study, we proposed a survey to students attending a Baccalaureate degree in Engineering, in the end of the first semester of 2020/2021. Due to the COVID-19 constraints, these students were taught using a hybrid regime, in which the practical classes were face-to-face and theoretical classes were online. It was applied an active learning teaching framework, encompassing Think-Pair-Share, Buzz sessions and case studies in the theoretical classes, and a modified version of eduScrum in the practical classes. We analyzed students' opinions on two dimensions of learning agility, viz. mental and people agilities. Globally, it is observed that students look for opportunities to develop new skills and knowledge, they enjoy delving into new information and apply past experience knowledge to future challenges. Moreover, they feel they are open-minded, tolerant and like helping others. This suggests a positive impact of active-learning in the development of learning agility.

## PROBABILITY and STATISTICAL METHODS: ASSESSING KNOWLEDGE and COMPETENCIES – case study at ISEC

Deolinda M. L. D. Rasteiro & Cristina M.R. Caridade

[dml@isec.pt](mailto:dml@isec.pt); [caridade@isec.pt](mailto:caridade@isec.pt)

Coimbra Institute of Engineering, Portugal

**Abstract:** The concepts taught during a Statistical Methods course make use of different mathematical skills and competencies. The idea of presenting a real problem to students and expect them to solve it from beginning to end is, for them, a harder task than just to obtain the value of a probability given a known distribution. Much has been said about teaching mathematics related to day life problems. In fact, we all seem to agree that this is the way for students to get acquainted of the importance of the contents that are taught and how they may be applied in the real world.

The definition of *mathematical competence* as was given by Niss (Niss, 2003) means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra- mathematical contexts and situations in which mathematics plays or could play a role. Necessarily, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills, in the same way as vocabulary, orthography, and grammar are necessary but not sufficient prerequisites for literacy. In the OECD PISA document (OECD, 2009), it can be found other possibility of understanding competency which is: *reproduction*, i.e, the ability to reproduce activities that were trained before; *connections*, i.e, to combine known knowledge from different contexts and apply them do different situations; and *reflection*, i.e, to be able to look at a problem in all sorts of fields and relate it to known theories that will help to solve it. The competencies that were identified in the KOM project (Niss, 2003, Niss & Højgaard, 2011) together with the three “clusters” described in the OECD document referred above were considered and adopted will slightly modifications by the SEFI MWG (European Society for Engineering Education), in the Report of the Mathematics Working Group (Alpers, 2013). At Statistical Methods courses often, students say that assessment questions or exercises performed during classes have has major difficulty to understand what is asked, meaning the ability to read and comprehend the problem and to translate it into mathematical language.

The study presented in this paper reflects an experience performed with second year students of Mechanical Engineering graduation of Coimbra Institute of Engineering, where the authors assessed statistical methods contents taught during the first semesters of 2017/2018 till 2020/2021 academic years. The questions assessment tests were separated into two types: ones that referred only to problem comprehension and its translation into what needed to be calculated and others where students need only to apply mathematical techniques to obtain the results. This paper is one of the results of RULES\_MATH project, [<https://rules-math.com/>], which aim was to develop tools to assess mathematical competencies.

Eight mathematical competencies identified are recognized in the assessment made to students in what concerns learning probability theory concepts. Since 2017 a study was carried out with Mechanical Engineering students at Coimbra Institute of Engineering. The results obtained cover the tests as a tool to assess competencies and its fitness to our students.

**TOPIC:How to assess competencies**

During the world pandemic of Covid 19, the need to transform teaching and learning methods was urgent and their impact on evaluation was also felt. As a result, this paper reflects the necessary adjustments on assessment that were proposed to students always having in mind the outcomes of Rules\_Math project.

Keywords: Assessment, Significant Learning, Competencies, Mathematics, Engineering

Mathematics Subject Classification: Primary 46N30, 47N30; Secondary 97B40, 97B50, 97C40, 97C50

**References:**

- [1] ALPERS, B. et al, A framework for mathematics curricula in engineering education, SEFI, 2013. Available online at: <http://sefi.htw-aalen.de/>
- [2] NISS, M., Mathematical Competencies and the Learning of Mathematics: The Danish KOM Project., In Proceedings of the 3rd Mediterranean, 2003.
- [3] NISS, M., & HØJAARD, T. (Eds.) (2011). Competencies and Mathematical Learning. Ideas and inspiration for the development of mathematics teaching and learning in Denmark, English Edition, Roskilde University.
- [4] NISS M., BRUDER R., PLANAS N., TURNER R., VILLA-OCHOA J.A. (2017) Conceptualization of the Role of Competencies, Knowing and Knowledge in Mathematics Education Research. In: Kaiser G. (eds) Proceedings of the 13th International Congress on Mathematical Education. ICME-13 Monographs. Springer, Cham
- [5] RASTEIRO D. D., MARTINEZ, V. G., CARIDADE, C., MARTIN-VAQUERO, J., QUEIRUGA-DIOS, A. (2018), "Changing teaching: competencies versus contents.", EDUCON 2018 - Emerging Trends and Challenges of Engineering Education". Tenerife, Spain.
- [6] RULES\_MATH Project, Project Erasmus+ 2017-1-ES01-KA203-038491" New Rules for Assessing Mathematical Competencies ". <https://www.researchgate.net/project/New-Rules-for-assessing-Mathematical-Competencies>.

# AN EDUCATIONAL NOTE ON THE 1D HEAT EQUATION

WIGAND RATHMANN

**Abstract.** The one dimensional (stationary) heat equation is very interesting from an educational point of view. The easiest formulation is

$$(1) \quad k \frac{d^2}{dr^2} T(r) = \dot{q}(r), \quad 0 \leq r_0 \leq r \leq r_1,$$

where the  $k$  denotes the conductivity of temperature and assumed to be constant. In the following we focus the Dirichlet boundary conditions:

$$(2) \quad T(r_0) = T_0 \text{ and } T(r_1) = T_1.$$

This model describes the stationary temperature distribution in a plate. By introducing the space dimension  $n = 1, 2, 3$  into the model the stationary temperature distribution in a plate ( $n = 1$ ), a cylindrical shell ( $n = 2$ ) and a sphere ( $n = 3$ ) are described in one model

$$(3) \quad k \frac{d}{dr} \left\{ r^{n-1} \frac{d}{dr} T(r) \right\} = \dot{q}(r), \quad 0 < r_0 \leq r \leq r_1.$$

This is an important model e.g. for process engineering students and is a perfect model to combine mathematical competencies with modelling knowledge. But what are the interesting aspects of this equation?

The ODEs can be seen as an operator equation

$$\mathcal{L}[T] = \dot{q}$$

for each  $n = 1, 2, 3$ . Starting from the linear operators view we have to discuss the solution of the homogenous equation

$$\mathcal{L}[T] = 0$$

and the corresponding fundamental systems depending on  $n$ . For all mentioned systems one can try to guess the two linear independent solutions or can compute the fundamental solutions analytically.

For each  $n = 1, 2, 3$  a different solution strategy can be used and motivated, like direct integration, separation concepts or the substitution for equations of Eulerian type. This can then be extended for some special inhomogeneities, where solution can be computed analytically.

One step ahead would be the numerical solution using finite differences. The 1d case is straight forward for central differences, while the two and three dimensional case has to deal with differences depending on the location. On the other hand the solvability of systems of linear equations can be discussed from the linear algebra view and the need of boundary conditions arising again. Since the analytical solutions are known the numerical results can be compared with them.

All this can be implemented in an interactive demonstration object running in a web browser and the students can compare the results obtained by themselves. The next steps could be now to consider boundary conditions of Neumann or Robin type or space depended conductivity of temperature.

**Keywords.** heat transfer, dynamic diagrams, ODE, numerics, linear algebra

(W. Rathmann) DEPARTMENT MATHEMATICS, FRIEDRICH-ALEXANDER-UNIVERSITÄT ERLANGEN-NÜRNBERG (FAU), APPLIED ANALYSIS, CAUERSTR. 11, 91058 ERLANGEN, GERMANY

*E-mail address*, W. Rathmann: wigand.rathmann@fau.de



# **Mathematical Competence Assessment and Work in Groups**

Daniela Richtarikova,  
Slovak University of Technology, Bratislava, Slovakia

## **Abstract**

The paper will present the results and experiences of pilot mathematical competence assessment within mathematical subjects at bachelor degree at the Faculty of Mechanical Engineering STU in Bratislava. It will introduce new methods for evaluation focussed on competence assessment developed by our teachers.

In addition, the paper will discuss experiences with not traditional assessment using two forms of group work pointing out its advantages and disadvantages. Dealing with nowadays pandemic situation and teaching in distance, we will present opinions and attitudes of our students comparing pros and cons, and moreover we will deal with forms of group work in online learning rated to be effective by our students.

Keywords: mathematical competencies assessment method, group work in assessment, group work in on line teaching and learning, technical university

Contact person: Daniela Richtarikova, [daniela.richtarikova@stuba.sk](mailto:daniela.richtarikova@stuba.sk)

## Purpose and Goal in Mathematics

1st February 2020

Hans Georg Schaathun <hasc@ntnu.no>

NTNU — Noregs Teknisk-Naturvitenskaplege Universitet

Most educators agree that we need to teach for understanding, rather than for rote learning. When we teach a mathematics degree, we expect the students to understand the relationships between different concepts, and understand why a given statement is true, and why given techniques work. When, in contrast, we teach service mathematics for engineering or other degree programmes, it is not obvious that the students need to understand in this sense. Certainly, very few students are motivated by this kind of understanding.

Given how difficult it appears to understand mathematics, we are often content, in service mathematics, if the students acquire a modest selection of computational skills. However, the German mathematics didactician Wolfram Meyerhöfer<sup>1</sup> claims that also students who struggle with mathematics can learn to understand, and moreover they *can only learn computational skills if* they understand.

Understanding seems to be sought, one way or another, by most people. As one of my students put it, «can't you just give us a formula, so that we can start to understand?» While this seemingly contradictory question puzzled me at the time, there may still be some sense behind it. In this presentation we shall review what we mean by understanding, in an aim to consolidate Meyerhöfer's view and this students question with our own experience with students struggling with service mathematics.

Some answers can be found in the Philosophy of Education, particularly in Hermeneutics, following Hans-Georg Gadamer and Peter Kemp. As Kerdeman<sup>2</sup> points out, *understanding* in the hermeneutic sense, does not refer to the mental process whereby we gain knowledge, as we do in mathematics through logical deductions. To Gadamer, understanding is a way of being. New understanding springs from the tension between the strange and the familiar in a continuous and iterative hermeneutic process. Gadamer speaks of «lived» understanding. The familiar in this process is not what we can rationally know, but what we have experienced, felt, and truly lived through.

The problem we meet when we teach mathematics as a purely abstract discipline, is that we ask the students to build a completely new system of knowledge, detached from their lived experience. The two systems of knowledge are so separated that no tension exist between the familiar experience and the mathematics, which is just alien and strange. Thus there is no scope for lived understanding, making the fundamental mathematics harder to understand, and even impossible for some

Therefore, I propose that, in the context of service mathematics, understanding should mean *to understand real world problems and situations in terms of mathematics*. This is, of course, essentially the mathematical thinking competency and the modelling competency in Mogens Niss' framework. However, we hold that this understanding is not a separate learning outcome to be pursued after basic training in mathematics. Rather, it is, except for students with a genuine prior interest and pleasure in mathematics in its own right, a necessary part of learning basic mathematics.

---

<sup>1</sup>«Ich will, dass jedes Kind rechnen lernt», *Der Spiegel*, 27.10.2013

<sup>2</sup>«Hermeneutics and Education: Understanding, Control, and Agency» in *Educational Theory*, volume 48, no. 2, 1998

# High Quality Questions for E-Assessment in Mathematics

Dennis Gallaun, Karsten Kruse, Christian Seifert  
Technische Universität Hamburg, Germany

January 17, 2020

E-assessment in mathematics can be used to efficiently provide a huge amount of problems for training. This makes it a valuable tool for large classes which are typical for service mathematics. In order to develop good questions, one can address different aspects, e.g.:

- randomisation of parameters such that all realisations are comparable for fairness,
- provide open-ended questions to demonstrate understanding rather than guesses,
- handling of errors, i.e. how to check for mistakes made by students.

Implementing randomisation of problems in a naive way may easily result in very easy as well as very difficult realisations of the same question. The goal of open-ended questions sometimes contrasts the automatic correction and therefore one portion of efficiency of e-assessment. Furthermore, correcting questions automatically puts some need to deal with mistakes made by students (e.g. whether it is just a typo or a real mistake while solving the problem). Of course, for e-assessment one needs to know the electronic system with its power as well as its pitfalls. In this talk we are going to describe how to deal with each of the aspects mentioned above.

Moreover, we will sketch the didactic scenarios in which we are using e-assessment in mathematics for large classes of engineering students.

# **Gamification in the study of mathematics for engineering students**

Marjeta Škapin Rugelj<sup>1</sup>, Jože Rugelj<sup>2</sup>

<sup>1</sup>*Faculty of Civil Engineering and Geodetics, University of Ljubljana, Slovenia*

<sup>2</sup>*Faculty of Education, University of Ljubljana, Slovenia*

Karl Kapp defined gamification is an emergent approach to instruction which facilitates learning and promotes motivation through the use of game elements, mechanics and game-based thinking. The student in gamified learning environment does not play an entire game, but participates in activities that include elements from games such as earning points, overcoming a challenge or receiving badges for completing tasks. The main idea is to integrate game-based elements into learning environments. Although this approach is essentially technology-independent, technology actually provides the most effective ways to implement it.

In addition to the elements of games, there are also a number of game dynamics or actions that take place while a player is engaged in a game. This includes merging, collecting/collecting, assigning resources, strategizing, building, solving puzzles, exploring, helping and role-playing . Combining these dynamics with the game elements mentioned above provides a context in which learners can be involved through gamification.

The idea is to collect the most effective game elements from a learning perspective and use them to motivate and engage learners.

Our students usually study for the exams using their notes. When they encounter a problem, they usually consult with peers and rarely seek help from teachers. To make exam preparation more efficient and more fun, we have introduced some gamification elements into the online learning environment. We designed and implemented an escape room as a quiz in the Moodle environment. The questions in the quiz were of different types and were intended to test their understanding of concepts and their computational skills.

The students divided into two groups that competed with each other. If a team manages to complete a certain number of tasks within a specified time, it is rewarded with bonus points. With these points students can improve their grades in the exam. Students usually solve computational problems on their own, but if they get different results, they talk about them a lot in the group until they find out the reason for the different results.

We plan to prepare a survey and ask the students about their opinion on the use of gamification elements in the learning process and their influence on the students' motivation.

## ROLE OF VISUALIZATION IN MATHEMATICS

D. Velichová, Slovak University of Technology (SLOVAKIA), [daniela.velichova@stuba.sk](mailto:daniela.velichova@stuba.sk)

Mathematics is one of those subjects, in which teaching requires illustrations of abstract theoretical concepts and relations in a suitable way on examples, with aim to help students in better understanding of connections and enable them to attain better insight to presented problems. Dynamic mathematical programs are educational tools enabling visualization of mathematical entities in the form of dynamic models. Direct interactive manipulations with models offer possibilities of heuristic approach in acquiring knowledge, while learning of theoretical data is directly connected with their practical application in the respective dynamic model. Development of model itself and also work with it during study require a new attitude to the role of teacher and student in the educational process, change of the form of educational environment and contents of the educational process.

Active learning methods and how to introduce these in engineering mathematics is a real pedagogical challenge. Mathematical subjects are mostly accepted as necessary assumption to be fulfilled in order to graduate successfully from some engineering study program, not as a useful tool and symbolic language or effective method for solving applied problems. Courses of basic mathematics are important for engineering studies in particular, because future engineers will be dealing with various situations requiring logical thinking, analytical reasoning and ability to transfer knowledge and generalise findings, in addition to quick calculation and decision-making skills, and experience with mathematical modelling of applied problems. Therefore, it is necessary to prepare engineering students for their future work already during their university studies.

Visualization can be regarded as a certain form of application, therefore development of visual models is also a kind of evaluation of the knowledge depth and level of understanding of the presented concept, fostering acquired knowledge and its usage, and its transfer into a different context. Development of dynamic models is also an inspiration how to utilize information technologies meaningfully in the role of didactic tool, which can not only attract learners, but also enable them to realize their own creative work. Both subjects of the educational process act in this didactic situation more as equal partners, opposite to the classical forms of didactic situations, where the role of teachers is active presentation of new facts and data, while role of learners is usually passive, just receiving presented facts. Active participation of students in educational process in interesting form can contribute to better understanding and more positive approach to learning itself, which becomes more a discovery of dependencies and investigation of activities and processes than memorizing of a huge amount of incomprehensible facts and not connected data. Dynamics opens way to discover connections, and to understand mutual dependencies, which is often more important than a detailed fragmented knowledge.

We will discuss some of new challenges of teaching by introducing active learning methods, present some examples of interactive applets in dynamic mathematical software and educational videos used in teaching basic courses of mathematics at technical university.

Keywords: didactics of mathematics, active learning, visualization, educational video, interactive applet

# Three-Level System for Teaching Mathematics in Engineering Education

Milena Sipovac<sup>1</sup>, Corinna Modiz<sup>1</sup>, Stefanie Winkler<sup>1</sup>, Andreas Körner<sup>1</sup>

<sup>1</sup>TU Wien, Institute of Analysis and Scientific Computing,  
Wiedner Hauptstraße 8-10, 1040 Vienna, Austria

{milena.sipovac, corinna.modiz, stefanie.winkler, andreas.koerner}@tuwien.ac.at

**Abstract.** Mathematics is the foundation on which the engineering studies are built on. Therefore, it is important to transfer the skills and the understanding of mathematics to the students in engineering in the very beginning of their studies. These courses are commonly known as ‘service courses’, and teaching these courses provides different challenges. One of the most common teaching obstacles is that the students of engineering studies are more interested to attend courses which are closely related to the subject of matter, which means that any courses which seem to have no direct connection to the topics of their field of studies can be discarded as not interesting. The second issue that occurs is that these service courses are bound to deliver very complex concepts in a very short time, since they usually take up to three semesters, so the teaching staff is faced with the necessity to teach in a compact, but still understandable manner. Additionally, other courses engineering students are confronted with, apply some advanced mathematical methods even before they are taught in the actual mathematics course.

In this contribution we will present a new approach of exercises to increase students’ motivation and at the same time support the different needs of heterogeneous groups. We will focus on the mathematics courses offered to students of physics, electrical engineering and geodesy and geoinformation. This mathematical course consists of lectures and a corresponding exercise course. In addition to attending the lectures and exercises, in order to get a positive grade on the course, the students need to pass three tests during the semester, as well as a final exam at the end of the semester.

A frontal lecture on the blackboard combined with weekly static examples is here considered as a traditional approach. The examples given each week are supposed to be prepared and presented in each exercise session. A percentage of students copy the results from their colleagues, without additional effort. In order to prevent this and to engage the students, in our more modern approach, each student gets their own set of examples to solve, so the static examples are replaced by randomised online examples. In this way a collaboration between the students is encouraged since they are prompted to talk about the examples instead of copying and pasting. These improvements lead to motivating a broader crowd of students to engage with the subject. Additionally, due to the randomisation of the variables and functions in the exercise problems, this online tool also provides support in the preparation phase for tests and the final exam, since the students have access to a great number of examples for practising and skill training. As constant participation of the audience is demanded, and a lecture is usually hard to follow, an online tool for visualisation of important concept is used during the lectures.

Our latest work introduced a modification in the exercise course, with the aim to allow students from different backgrounds to recognize their skill level and to improve from their current position. This was accomplished by separating the examples given each week into three different levels. The first level contains examples which cover the basics of the topic and represent the foundational skills necessary to proceed. Each example of this level must be solved correctly in order to pass the exercise course. The exercises from this level usually contain examples from previous lectures and sometimes also knowledge that should have been taught in school. Level two consists of examples which are based on the current topics from the lecture, and they require a combination of the already attained skills, as well as learning new skills to find the right solution. A certain percentage of these examples must be solved in order to pass the exercise course, but it is not required to solve every example of this level. The last level describes the so-called master class. Here, the students are asked to create their own example related to a certain topic. To succeed they have to formulate the example and provide the according solution. Each week there is one or two problem definitions of this kind. Creating an example by themselves requires a good understanding of the subject matter and additionally students are supposed get creative and think more about the topic and its methods, compared to solving the already given problems. In the end, the students are required to collect a total of 60% of the possible points by summing the points of the exercises of each week.

In the paper, we will report on heuristics and show some results of the trial phase of this format. Examples of each level will be presented. An analysis of the results and the student’s engagement will be performed.

**Keywords:** Blended Learning, heterogeneous student groups, motivation, competences

**Subject:** The goal of teaching – “Service mathematics – mathematics for non-specialists”.

## **An investigation into variations of engineering students' attitude towards mathematics across gender and age: A MIMIC model approach**

Yusuf Zakariya, University of Agder

Students' attitude towards mathematics is a crucial affective construct that has been associated with students' competence development on mathematics tasks and performance. Researchers across the world have focussed attention on the relationship between students' attitude towards mathematics and learning outcomes in the subject. There seems to be a scarcity of careful studies on the psychometric properties of attitude towards mathematics scales. The available studies are limited to establishing factor structures of the scales and computing reliability indices. The present study is a follow-up research on the validity of an attitude towards mathematics questionnaire that has been in use for over two decades in Norway. The purpose of the study is twofold. First, to confirm the factor structures of the questionnaire in an independent sample from the initial study. Second, to investigate the variations of the questionnaire across gender and age of first-year engineering students. In specific terms, the following questions are raised: Do the five items of attitude towards mathematics questionnaire measure a single factor? Does the attitude towards mathematics vary across students' gender? Does the attitude towards mathematics vary across students' age? To address these questions, data are generated using a students' attitude toward mathematics questionnaire and the generated data are analysed using a Multiple Indicators, Multiple Causes (MIMIC) structural equation modeling technique. The results confirm a single factor structure for the five-item questionnaire. As an improvement of the previous study that recommends two error covariances between some items of the questionnaire, only one error covariance is found to be sufficient to achieve the single factor structure of the questionnaire. Further, evidence from the present study shows that the attitude toward mathematics questionnaire is neither gender-biased nor age-biased. As such, the questionnaire can be used to measure engineering students' attitude towards mathematics regardless of the gender and age of the students.